Interfacial crack-tip constraints and $J$-integrals in plastically mismatched bi-materials

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Abstract

The effect of the $T$-stress and plastic mismatch on interfacial crack-tip stress under small scale yielding condition is investigated in this paper via detailed finite element analyses. The mismatch in plastic strength/hardening is focused, and plane strain elastic-plastic crack-tip fields have been modeled with modified boundary layer formulation. Plastic mismatches as well as compressive $T$-stresses in bi-material are shown to affect the interfacial crack-tip constraint substantially. Simple patching of slip-line fields is presented to characterize the interfacial crack-tip stress fields. Effects of $T$-stress and plastic hardening mismatch on asymmetry of the $J$-integral for bi-materials, and its implication on interfacial toughness are also discussed. © 2001 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Assessing significance of a defect within the weld, including that at the heat affected zone (HAZ), is an important task of structural integrity assessments. In particular, the crack at the HAZ (interface) is important in many practical problems, due to the fact that the HAZ is the most susceptible region for cracking, which is the main subject of this paper. For interface cracks, the mismatch in elastic as well as plastic properties can occur. In typical welded steel structures the mismatch in elastic properties of the weld metal and the base plate can be neglected even for dissimilar metal welds, but a significant mismatch in plastic properties (such as the yield strength and the strain hardening) can occur. Such mismatch in plastic properties will affect deformation and consequently fracture behaviors of welded joints (see e.g. Refs. [1,2]).

The strength mismatch effect on the crack driving force, such as the $J$-integral and the crack-tip opening displacement, for strength mismatched structures can be quantified by simply incorporating the mismatch effect on the plastic yield load (mismatch yield load) [3–9]. Numerous finite element validations [9,10] as

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well as experimental validations [11] support the proposed crack driving force estimation scheme for strength mismatched structures. Note that such scheme can be applied to any crack location within the weld, including that at the interface (HAZ), as long as the corresponding mismatch yield load is used (see e.g. Ref. [11]).

However, the mismatch in plastic properties also affects local stresses and thus fracture toughness of welded joints, which has been demonstrated by fracture behavior of actual welds. For instance, Kocak et al. [12] found that the HAZ toughness could take quite different magnitudes, and could be improved by reducing the degree of over match of the weld metal. There have been several works in the literature, regarding the strength mismatch effect on interfacial crack-tip constraint. Zhang and co-workers [13,14] proposed the $J-m$ theory where the parameter $m$ reflects the constraint due to the mismatch in plastic properties, in contrast to the $Q$-parameter reflecting the constraint effect due to the geometry and the mode of the loading [15]. The $m$-parameter was determined in the similar fashion to the $Q$-parameter, namely from elastic–plastic finite element (FE) analyses. With enormous amount of elastic–plastic FE analyses involving many variables such as geometry, mismatch in yield strengths and in strain hardening, they have provided a fundamental way to treat the interfacial crack-tip stress fields. Ganti et al. [16] proposed the strength mismatch effect on asymptotic interfacial crack-tip fields, in the context of the perfectly plastic material. The constraint effect due to the geometry and the mode of loading has been also investigated in terms of the elastic non-singular $T$-stress. They extended the analysis to the plastically hardening material, assuming two materials have the same hardening capacities (without the hardening mismatch). The above works are limited to remote mode I loading. Recently, the strength mismatch effect on asymptotic interfacial crack tip fields under $K$-dominant, mixed mode loading has been reported by Sham et al. [17]. This work is again based on the assumption of perfectly plastic solids.

The present work focuses on the study of the mismatch effect not only in plastic yield strength but also in strain hardening on asymptotic interfacial crack-tip fields. First the effects of $T$-stress and strength/hardening mismatches on the plastic zone size at the crack tip are examined via elastic–plastic FE analyses (ABAQUS [18], 1998) in plane-strain small scale yielding (SSY) conditions. Then stress triaxialities at the bi-material interfacial crack tip are also measured by FE analyses. Based on these FE results, the effects of $T$-stress and plastic property mismatches on the stress triaxiality at the interfacial crack tip are presented quantitatively, and interpreted in terms of slip-line fields. Finally asymmetry of the $J$-integral at the interfacial crack-tip and its implication on the interfacial toughness of bi-material are discussed.

2. Finite element analyses

2.1. Finite element model

Consider a plane strain interface crack lying between two elastic–plastic materials subject to mode I loadings under small scale yielding conditions. Two materials in such dissimilar joint can have different thermal, elastic, and plastic strength as well as strain hardening properties. For simplicity, this paper concentrates only on plastic mismatches, i.e., two materials have the same elastic and thermal properties but different strength/hardening properties, as schematically depicted in Fig. 1a and b. In case of plastic strength mismatch, we consider elastic–perfectly plastic solids with different yield strengths. For consistent notations throughout the paper, assume that the material 1 is a reference material, and is always plastically weaker than the material 2 (has a lower yield strength, $\sigma_1 = \sigma_{LY} \leq \sigma_2 = \sigma_{HY}$). Moreover, the subscript “0” refers to the property of reference material, i.e., $\sigma_1 = \sigma_0 = \sigma_{LY}$. Plastic strength mismatch of two materials is characterized by the plastic strength mismatch $M$ defined by
Fig. 1. (a) Strength mismatch ratio $M$ and (b) Rice and Rosengren type power law hardening materials.

$$M \equiv \frac{\sigma_2}{\sigma_1} \left( = \frac{\sigma_2}{\sigma_0} \right),$$

which ranges from 1 (for homogeneous material) to $\infty$ (for bi-materials consisting of elastic–perfectly plastic material bonded to an elastic material). In cases of strain hardening mismatched materials, the tensile properties of materials are assumed to follow elastic, power law hardening as:

$$\frac{\varepsilon}{\varepsilon_0} = \begin{cases} \frac{\sigma}{\sigma_0} & \text{for } \sigma \leq \sigma_0, \\ (\frac{\sigma}{\sigma_0})^n & \text{for } \sigma > \sigma_0; \ 1 < n < \infty. \end{cases}$$

Here $\sigma_0$ is tensile yield strength of the reference elastic–perfectly plastic material, $\varepsilon_0 = \sigma_0/E$ is yield strain and $n$ is strain hardening exponent. Fig. 1b shows various stress–strain curves for linear elastic materials with $n = 1$, strain hardening materials with $n = 5, 10$ and for elastic–perfectly plastic materials with $n = \infty$, being used in our analyzes. In these cases of hardening mismatch, with four homogeneous materials and six bi-materials matched with those four materials, total 10 FE models can be prepared. By adopting Eq. (2), we can define obvious linear elastic region, and then accordingly define the “identical” yield strength, in contrast to the Ramberg–Osgood model.

Small strain theory was used, and the elastic–plastic material was modeled as an isotropic elastic–plastic material that obeys $J_2$ flow theory. To avoid problems associated with incompressibility, 8-node reduced integration elements (element type CPE8R from the ABAQUS library [18]) were adopted. Plane strain crack-tip deformation was modeled by modified boundary layer formulations (MBL) using focused meshes of the type shown in Fig. 2. The meshes typically involved about 1500 elements consisting of 40 rings and 34 elements concentric with the crack tip. The crack tip thus comprised 69 independent but initially coincident nodes. Mesh refinement was such that the radius of the first ring of elements was less than one-millionth of the radius of the most outer ring ($=R$).

2.2. Modified boundary layer formulation

Plane strain FE analyzes were performed with the modified boundary layer formulation based on the first two terms of Williams expansion

$$\sigma_{ij} = \frac{K_1}{\sqrt{2\pi r}} S_j(\theta) + T \delta_{ij} \delta_{ij},$$

(3)
In Eq. (3), $s_{ij}(\theta)$ are the angular variations of the stress components, and the second term corresponds to the non-singular $T$-stress term. In actual FE analyses, in-plane displacement boundary conditions were applied on the outer boundary of the domain shown in Fig. 2.

$$u_i = \frac{K_1}{E} \sqrt{\frac{r}{2\pi}} f_i(\theta, v) + \frac{T}{E} r g_i(\theta, v).$$

(4)

Here $E$ is the Young’s modulus, $v$ is the Poisson’s ratio and $(r, \theta)$ are cylindrical coordinates centered at the crack tip. The function $f_i(\theta, v)$ are the angular variations of the cartesian displacement components ($i = x, y$) of the plane strain elastic singular field, and $g_i(\theta, v)$ are the angular variations of the displacements due to the (plane strain) $T$-term. The exact expressions for $f_i(\theta, v)$ and $g_i(\theta, v)$ are listed in Table 1.

In so far as the two-parameter characterization is valid, the crack-tip fields of the MBL solution far from the outer boundary and outside the crack-tip blunting zone represent those of any crack with the same values of $K_1$ and $T$. Elastic-plastic crack-tip fields were obtained by systematically varying $\tau$ ($\equiv T/\sigma_0$),

<table>
<thead>
<tr>
<th>Fields</th>
<th>$x$-Component</th>
<th>$y$-Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_i$</td>
<td>$(1 + v)(3 - 4v - \cos \theta) \cos(\theta/2)$</td>
<td>$(1 + v)(3 - 4v - \cos \theta) \sin(\theta/2)$</td>
</tr>
<tr>
<td>$g_i$</td>
<td>$(1 - v)(1 + v) \cos \theta$</td>
<td>$-v(1 + v) \sin \theta$</td>
</tr>
</tbody>
</table>
while keeping $K_1$ constant. The maximum value of $\tau$ that can be applied is determined by the plastic yielding condition. Parks [19] showed that the applied $\tau$ should satisfy to guarantee that some portion of the domain would remain elastic. For $v = 0.3$, Eq. (5) gives that $|\tau| < 1.125$. But for SSY condition, the value of $|\tau|$ should be less than this. Hence we obtained solutions for $|\tau|$ up to 0.7 with an increment 0.1. Since the elastic displacement boundary was applied on the outer boundary, the plastic zone should be limited to a small fraction of the entire domain to be consistent with boundary conditions. Consequently, at $|\tau| = 0.7$, the maximum radius of plastic zone from the crack tip was about $0.01R$, which implies that the SSY condition was sufficiently fulfilled.

$$|\tau| < \frac{1}{\sqrt{1 - v + v^2}}. \tag{5}$$

3. Plastic zone

3.1. Effect of yield strength mismatch

Consider a pure $K_1$ loading (zero $T$-stress) with varying $M$. Writing the maximum radius of the plastic zone in each side of the two materials as $(r_p)_L = A_L(K_1/\sigma_{LY})^2$ and $(r_p)_H = A_H(K_1/\sigma_{HY})^2$, the effect of $M$ on $A_L$ and $A_H$ is shown in Fig. 3a. In the FE model, the plastic zone in each material was defined as the zone where the equivalent Mises stress $\sigma_e$ exceeds 99\% of the yield strength of the respective material, i.e., $\sigma_e \geq 0.99\sigma_{LY}$ and $\sigma_e \geq 0.99\sigma_{HY}$ respectively. As $M$ increases, $A_L$ increases slightly from $\sim 0.16$ to $\sim 0.18$, whereas $A_H$ decreases rapidly from $\sim 0.16$ to zero. Thus the effect of $M$ on plastic zone size is negligible in the lower strength material but is significant in the higher strength material. To investigate the effect of the $T$-stress, two limiting values of $M$ are considered: $M = 1$ and $M = \infty$. The $T$-stress in the subsequent sections is normalized with respect to $\sigma_{LY} = \sigma_0$. Fig. 3b compares the dependence of $A_L$ on $\tau$ ($\equiv T/\sigma_0$) for $M = 1$ and $M = \infty$. The results suggest that the plastic zone size in the lower strength material of dissimilar joints remains almost constant regardless of $M$. Of course, its size in the higher strength material strongly depends on $M$ as shown in Fig. 3a.

![Fig. 3. Variation of plastic zone size with (a) the strength mismatch $M$ and (b) $T$-stress.](image)
3.2. Effect of strain hardening mismatch

Effect of various $T$-stresses on plastic zone size $r_p$, for three elastic–plastic homogeneous materials with strain hardening exponent $n = 5, 10, \infty$ is shown in Fig. 4a. For homogeneous materials, given $K_I$ and $T$-stress values, it is shown that the plastic zone size becomes larger with less strain hardening. However, the effect of $T$-stress on plastic zone size is dominant, while the effect of strain hardening on the plastic size is relatively small. Fig. 4b shows the dependence of plastic zone size upon $T$-stress and strain hardening mismatch in the bi-material. Each data are plastic zone size measured in elastic–perfectly plastic material that is joined with another different strain hardening material (see Fig. 1b). It is observed that in case of bi-material, the plastic zone size in elastic–perfectly plastic material is almost independent of strain hardening property of another joining material, as in the case of strength mismatched bi-material. The plastic zone size becomes least when $\tau = 0.2$, and plastic zone size increases as $|\tau|$ increases, and plastic zone size in the absolute value of negative $\tau$ increases rapidly. The plastic zone size of elastic–perfectly plastic material can be described as a function of $\tau$ (solid line of Fig. 4b).

$$r_p/(K_I/\sigma_0)^2 = 0.124 - 0.160\tau + 0.193\tau^2 - 0.234\tau^3; \quad -0.7 \leq \tau \leq 0.7.$$ \hspace{1cm} (6)

4. Crack-tip stress fields

4.1. Effect of strength mismatch

4.1.1. Stress triaxiality at the crack tip

Consider first bi-material systems consisting of two elastic–plastic materials with plastic strength mismatch of $M$, subject to pure $K_I$ boundary conditions, i.e., zero $T$-stress. For interfacial crack problems, opening stress at the interface ($\theta = 0$) would be of most interest, since fracture (either ductile rupture or brittle debonding) can take place along the interface due to imperfections or weak bonding there. The opening stress at the interface can be approximated in terms of $M$ as
(σ_m)_{θ=0}/σ_0 = (1 + 13π/9 - 2φ_1)/√3, \quad (7a)

(σ_{22})_{θ=0}/σ_0 = \cos[2(-φ_1 + π/4)]/√3 + (σ_m)_{θ=0}, \quad (7b)

where φ_1(M) = -0.314\pi(M - 1) + π/4 for 1 ≤ M < 1.8, and φ_1(M) = 0 for M ≥ 1.8. The parameter φ_1 reflects the angular size of the constant stress sector at the crack tip. Resulting variation of (σ_m)_{θ=0}/σ_0 and (σ_{22})_{θ=0}/σ_0 with M are compared with the FE results in Fig. 5. When M ≥ 1.8, the shear stress at the interface becomes (σ_{22})_{θ=0} = k = σ_0/√3, in contrast to the homogeneous materials where (σ_{22})_{θ=0} = 0. Increasing plastic strength mismatch is associated with higher crack-tip constraints. The value of (σ_{22})_{θ=0} for M ≥ 1.8 is about 11% higher than that for M = 1. Moreover, the value of σ_m for M ≥ 1.8 is about 40% higher than that for M = 1. Therefore plastic strength mismatch would play a significant role in interfacial crack-tip constraints. It is worth noting that the slip-line analysis for yield strength mismatch by Ganti et al. [16] showed that the asymptotic stress field in the weaker material should be constant for M ≥ 1.41. On the other hand, our results show that this limit is not reached before M ≥ 1.8. This may be due to the fact that the MBL model used in the present work was for the case of T = 0, where plasticity does not surround the crack tip. Another possible reason is that the stress values were evaluated at a distance from the crack tip.

Consider bi-material systems consisting of two elastic–plastic materials with plastic strength mismatch of M, subject to pure K_1 boundary conditions. At positive τ(>0), plasticity is again fully developed at all angles of θ, and the fan is fully extended up to the interface. Such field for bi-materials is analogous to the Prandtl field for homogeneous materials, and thus the stresses from such field are the highest values achievable for interfacial cracks in bi-materials. Applying the Hencky equation with the boundary condition of σ_m = σ_0/√3 at θ = 180°, such field provides

(σ_m)_{θ=0}/σ_0 = (σ_{22})_{θ=0}/σ_0 = 3.30; \quad (σ_{22})_{θ=0}/σ_0 = 1/√3. \quad (8)

Note that the values of (σ_m)_{θ=0}/σ_0 = 3.30 and (σ_{22})_{θ=0}/σ_0 = 3.30 are 38% and 11% higher respectively than the Prandtl values for homogeneous materials. For τ(<0), the effect of the T-stress on the hydrostatic stress and tensile stress at the interface can be simply expressed as follows.

![Fig. 5. Variation of the hydrostatic stress and the tensile stress at the interface, (σ_m)_{θ=0} and (σ_{22})_{θ=0}, with plastic strength mismatch M.](image-url)
For $M = \infty$
\[
(\sigma_m)_{\theta=0}/\sigma_0 = 3.20 + 0.56\tau - 1.97\tau^2, \tag{9a}
\]
\[
(\sigma_{22})_{\theta=0}/\sigma_0 = 3.20 + 0.29\tau - 1.60\tau^2. \tag{9b}
\]
For $M = 1$ (elastic-plastic homogeneous)
\[
(\sigma_m)_{\theta=0}/\sigma_0 = 2.290 + 0.712\tau - 1.1\tau^2, \tag{10a}
\]
\[
(\sigma_{22})_{\theta=0}/\sigma_0 = (\sigma_m)_{\theta=0}/\sigma_0 + 1/\sqrt{3}. \tag{10b}
\]

Resulting approximations are compared with the FE result in Fig. 6. As noted, $\tau \geq 0$, values of $(\sigma_m/\sigma_0)_{\theta=0}$ for $M = \infty$ can be about 40% higher than those for homogeneous materials. For negative $\tau$, the difference can be as much as 50% at $\tau = -0.7$.

4.1.2. Characteristics of strength mismatch induced constraints

In this section, we explain the change of stress field at the interfacial crack tip with slip line field theory. The Prandtl field is based on the assumption that plasticity completely surrounds the crack tip. On this basis the stresses can be solved starting from the traction-free crack surface region denoted I (135° ≤ $\theta$ ≤ 180°) in Fig. 7. In this region the yield criterion and the free surface require that the stress field is a homogeneous tensile (or possibly compression) field parallel to the crack flanks. It is convenient to work in cylindrical coordinates $(r, \theta)$ centered at the crack tip such that the crack lies along the $\theta = \pi$ axis. The stress field in region I can now be written in terms of yield strength in shear, $k = \sigma_0/\sqrt{3}$, as
\[
\sigma_r = k(1 + \cos 2\theta), \tag{11a}
\]
\[
\sigma_\theta = k(1 - \cos 2\theta), \tag{11b}
\]
\[
\sigma_\tau = k \sin 2\theta, \tag{11c}
\]
\[ \sigma_{zz} = \sigma_m = k. \]  

The stresses in the rest of the field can be deduced from the Hencky equations (\( d\sigma_m = -2k \, d\theta \)), which express the equilibrium requirements in terms of the rotation of the slip lines. The straight slip lines in region I thus imply a homogeneous stress state in the triangle. Following a slip line into the centered fan, denoted II \( (45^\circ \leq \theta \leq 135^\circ) \) in Fig. 7, gives the stress state in this region

\[ \begin{align*}
\sigma_{\phi\theta} &= \sigma_{rr} = \sigma_{zz} = \sigma_m = k(1 + 3\pi/2 - 2\theta), \\
\sigma_{r\theta} &= k.
\end{align*} \]  

Finally, in the diamond ahead of the crack, denoted III \( (0^\circ \leq \theta \leq 45^\circ) \) in Fig. 7, the stress terms consist of the homogeneous stress state as

\[ \begin{align*}
\sigma_{\phi\theta} &= k(\pi + 1 + \cos 2\theta), \\
\sigma_{rr} &= k(\pi + 1 - \cos 2\theta), \\
\sigma_{zz} &= \sigma_m = k(1 + \pi), \\
\sigma_{r\theta} &= k \sin 2\theta.
\end{align*} \]

We now summarize schematically, as shown in Fig. 8, how the \( T \)-stress and hardening mismatch perturb the above Prandtl field of Eqs. (11)–(13). For negative \( T \)-stress, the line \( l \) in elastic–perfectly plastic material rotates clockwise which decreases the fan angle, and interfacial crack-tip constraint is accordingly relieved. The positive \( T \)-stress brings a counteraction. If higher strength material is joined, the line \( m \) in elastic–perfectly plastic material rotates clockwise which increases the fan angle, and interfacial crack-tip constraint accordingly increases. Similar discussion has been given by Ganti et al. \[16\]. More details are given below in the Section 4.2.2 of hardening mismatched bi-materials.

### 4.2. Effect of strain hardening mismatch

#### 4.2.1. Stress triaxiality at the crack tip

Fig. 9a–d presents the change of crack-tip stress triaxiality \( (\sigma_m/\sigma_0) \) with angular position at a distance \( r = 2J/\sigma_0 \) from the crack tip. Each figure shows the stress triaxiality in elastic–perfectly plastic material for various values of \( \tau \), when that base elastic–perfectly plastic material is joined to the other materials of different strain hardening properties. Fig. 9a shows the result for the bi-material consisting of linear elastic material and the base elastic–perfectly plastic material, which is the most extreme strain hardening mismatch, while Fig. 9d shows the result for the homogeneous elastic–perfectly plastic materials, which is...
another extreme condition. Fig. 9d shows that crack-tip constraint \( \sigma_m/\sigma_0 |_{\theta=0} = 2.39 \) for the homogeneous elastic–perfectly plastic material with \( \tau > 0 \) is in accordance with the Prandtl solution having the highest values in small scale yielding condition. For the bi-material, however, higher crack-tip constraint is observed for \( \tau > 0 \) due to strain hardening mismatch. The value of \( \sigma_m/\sigma_0 |_{\theta=0} = 3.39 \) in Fig. 9a is the highest constraint state for the SSY bi-material and is 38% higher than the Prandtl solution. These are two extreme strain hardening cases, and interfacial crack-tip constraint for bi-materials with intermediate strain hardening mismatch exists between these two extreme cases (Fig. 9b and c). Fig. 9d indicates that stress triaxiality at the crack tip \( (\sigma_m)_{\theta=0}/\sigma_0 \) of \( -\tau = 0.7 \) is 25% lower than that of \( \tau = 0 \), and that stress triaxiality at the crack tip of \( +\tau = 0.7 \) is 5% higher than that of \( \tau = 0 \). As the strain hardening mismatch increases, stress triaxialities of positive \( \tau \) slightly vary, while stress triaxialities with negative \( \tau \) continue to change substantially (Fig. 9a–c).

Fig. 10 shows the variation of stress triaxialities \( (\sigma_m/\sigma_0 |_{\theta=0}) \) with T-stress at a distance \( r = 2J/\sigma_0 \) from the crack tip in the reference elastic–perfectly plastic material in bi-materials consisting of that reference elastic–perfectly plastic material and another different strain hardening materials. The bi-material with \( n = 1 \), which consists of linear elastic material and the base elastic–perfectly plastic material, is the most extreme strain hardening mismatch. The material with \( n = \infty \), which means a homogeneous elastic–perfectly plastic material, is another extreme case. The following two features are noteworthy in the figures. First, \( \tau \) in a bi-material have a serious effect on crack-tip constraint condition likewise in homogeneous material. Second, more importantly, the bi-material carries higher stress constraint than homogeneous material for all \( \tau \). As observed in the figures, stress triaxiality of bi-material is about 40–50% higher than that of homogeneous material. This feature is clearly due to strain hardening mismatch. In case of bi-materials of intermediate strain hardening mismatch \( (n = 5, 10) \), the constraint states locate between these two extreme values. Therefore, for the bi-material consisting of elastic–perfectly plastic material and ma-
Fig. 9. Normalized hydrostatic stresses at various values of $T$-stress ($\tau = T/\sigma_0$) and plastic hardening mismatch.

terial of general strain hardening exponent, stress triaxiality at the interfacial crack tip (more exactly, in elastic–perfectly plastic material) can be obtained by the following equation (14) as a function of $T$-stress and the strain hardening exponent $n$ of the joining material.

\[
\left(\frac{\sigma_m}{\sigma_0}\right)_{\tau=0} = A_n + B_n \tau + C_n \tau^2 + D_n \tau^3; \quad -0.7 \leq \tau \leq 0,
\]

\[
\begin{align*}
A_n &= 2.11 + 1.50n^{-1} + 5.69n^{-2} - 6.18n^{-3}, \\
B_n &= 0.68 - 1.07n^{-1} - 15.07n^{-2} - 13.63n^{-3}, \\
C_n &= -0.88 + 2.14n^{-1} - 4.07n^{-2} + 1.49n^{-3}, \\
D_n &= 0.29 - 1.11n^{-1} - 1.46n^{-2} - 0.45n^{-3}.
\end{align*}
\tag{14}
\]

Fig. 11 demonstrates the accuracy of the above function for negative $T$-stress and the given strain hardening mismatches ($n = 1, 5, 10, \infty$). Validity of the above equation for other strain hardening
Fig. 10. Normalized interfacial hydrostatic stresses for various values of $T$-stress and plastic hardening mismatch.

Fig. 11. Normalized interfacial hydrostatic stresses obtained from Eq. (6) and FEM for various values of normalized $T$-stress and plastic hardening mismatch.

mismatches ($n = 7, 13$) is also confirmed in Fig. 11. Note the shaded marks from FE analyzes and the corresponding prediction lines by Eq. (14).

4.2.2. Characteristics of hardening mismatch induced constraints

In this section, we explain the change of stress field at the interfacial crack tip with slip-line field theory, and show schematically how the $T$-stress and hardening mismatch perturb the above Prandtl field of Eqs. (11)–(13). Fig. 12a–c shows stress components with changing $T$-stresses ($T/\sigma_0 = \tau = 0.7, 0, -0.7$). It is shown that right side boundary of fan, the line $m$, does not change (fixed at 45°), and left side boundary, the line $l$ only changes. For $\tau = 0.7$, fan increases clockwise sufficiently, so the line $l$ reaches its critical angle 135° (Fig. 12a), consisting with Du and Hancock’s study [20]. On the other hand, when $\tau < 0.446$ [20], the line $l$ rotates clockwise, and crack-tip constraint accordingly decreases. When $\tau = 0$, the line $l$ has an angle of 128°, and when $\tau = -0.7$, the line $l$ has an angle of 78° (Fig. 12b and c). This effect of $T$-stress is similarly
Fig. 12. Normalized crack-tip stresses at various values of $T$-stress for elastic–perfectly plastic homogeneous material.
observed in bi-material likewise in homogeneous material. Fig. 13a–d shows that the strain hardening mismatch of bi-material changes the angular position of the right side boundary of fan, the line \( m \) only, for given \( \tau = 0.7 \) (i.e., when \( T \)-stress is fixed as a value giving the angle of the line \( l = 135^\circ \)). That is, for the elastic–perfectly plastic material, the line \( m \) has the angle of \( 45^\circ \), and as higher strain hardening material is joined to it, the line \( m \) in the reference elastic–perfectly plastic material rotates clockwise and becomes to have angles of \( \theta_{\text{m}} = (30^\circ, 16^\circ, 0^\circ) \) for joined materials of \( n = (10, 5, 1) \) respectively. Further the shear stress in bi-material has nonzero value at the interface, differently from that of homogeneous material. All these features are schematically illustrated in Fig. 14.

We may summarize the effect of \( T \)-stress and strain hardening mismatch on the fan region of the reference elastic–perfectly plastic material as follows. For negative \( T \)-stress, the line \( l \) in elastic–perfectly plastic material rotates clockwise which decreases the fan angle, and interfacial crack-tip constraint is accordingly relieved (Figs. 12a–c and 14a). The positive \( T \)-stress brings a counteraction. If higher strain hardening material is joined, the line \( m \) in elastic–perfectly plastic material rotates clockwise which increases the fan angle, and interfacial crack-tip constraint accordingly increases (Figs. 13a–d and 14b). These phenomena in hardening mismatched bi-materials are analogous to the characteristics of interfacial crack-tip constraint in non-hardening bi-materials consisting of different yield strength materials in Section 4.1.2 (also see Fig. 8). Consequently yield strength mismatch and strain hardening mismatch have a qualitatively equivalent effect on SSY crack-tip stress field. It is known that the change in stress field due to hardening mismatch is somewhat more local (more rapidly decreasing as one moves away from the crack tip) com-
pared to the one resulting from yield stress mismatch. Thus, the change in stress field is more strongly depending on the position at which this value is calculated in the former case.

5. Discussion

In this section, the effect of the strain hardening mismatch on the bi-material fracture behavior is interpreted in view of crack driving force and fracture toughness of welded joints. We first investigate the contribution of each constituent material of a strain hardening bi-material to the crack-tip fracture governing parameter J-integral, a key parameter describing the crack-tip stress and strain fields in elastic-plastic fracture mechanics. As presented schematically in Fig. 15, the total J-integral in bi-material can be decomposed into \( J_L \) from the lower strain hardening material (higher strain hardening exponent) and \( J_H \) from the higher strain hardening material (lower strain hardening exponent) as follows.

\[
J = J_L + J_H. \tag{15}
\]

Fig. 16 shows the change of J-integral with the distance from crack tip in four strain hardening homogeneous materials. Here \( J_{SEAR} \) is obtained directly from finite element analysis using virtual crack extension method [21] or domain integral method [22], and \( J_{FAR} \) is calculated from the preset value of \( K_I \) on the modified boundary layer, i.e., \( J_{FAR} = (1 - v^2)K_I^2/E. \) As strain hardening decreases, J-integral near the crack tip is observed to decrease by maximum 13% \((n = \infty)\). The reason is that isotropic elastic–plastic material obeying \( J_2 \) incremental plastic theory is used instead of deformation theory of plasticity on which J is based. Fig. 17a shows the characteristics of J in a strain hardening bi-material. When linear elastic material \((n = 1)\) is joined to elastic–perfectly plastic material \((n = \infty)\), \( J_L \) increases dramatically as the domain is getting close to interfacial crack tip, but \( J_H \) decreases rapidly. Fig. 17b describes the contribution
of $J_\text{L}$ to $J$, when different strain hardening materials ($n = 1, 5, 10$) are joined to elastic–perfectly plastic material ($n = \infty$). It is shown that as the domain is getting close to interfacial crack tip, and as strain hardening of joining material becomes higher, the portion of $J_\text{L}$ increases (especially higher than 0.9 for $n = 1$). Fig. 18a demonstrates the effect of $T$-stress on $J$-integral values at the interfacial crack tip, $r = 2J/\sigma_0$. For $n = 1, 5, 10$, $J$-integral has almost constant value despite changing $T$-stress, and only for
$n = \infty$, $J$-integral with negative $T$ decreases as much as 18% compared to that with positive $T$. $J_L/J$ values near interfacial crack tip in strain hardening bi-materials vary within 10% with changing $T$-stress as given in Fig. 18b. The features observed in Figs. 17 and 18 indicate that $J_L/J$ is dominantly determined by the strain hardening exponents of joining materials rather than $T$-stress.

In Section 4, we noticed that triaxiality of the lower strength/hardening material (=higher toughness material) in plastically mismatched bi-material is higher than that of the homogeneous case of lower strength material itself. Moreover, in this section, we also observed that asymmetry of $J$-integral results in a higher value of $J_L$ in the lower (strength)/hardening material side. Combining these two means that the lower strength/hardening side carries higher crack driving force $J$ and higher crack-tip constraints, which suggest that the crack near interface could grow in the lower strength/hardening material in spite of its higher toughness than that of joining high strength/hardening material. Experimental evidence of this can
be found in Ref. [23] where the (near) interface crack located in the harder material of Ti–6Al–4V deviates into the softer (but much tougher) material of pure Ti.

6. Conclusion

In this paper, interfacial crack-tip stress fields of plastically mismatched bi-materials under mode I SSY loading were investigated in plane strain condition via FE analysis, based on modified boundary layer method. It was observed that plastic zone size in plastically mismatched bi-material is affected with $T$-stress dominantly, but is not dependent on the mismatches in plastic properties of joining materials. Likewise homogeneous materials, geometric shape and load condition have a serious effect on stress triaxiality at the crack tip in bi-material at first. It was then shown that interfacial crack-tip stress in plastically mismatched bi-material is maximum 50% higher than homogeneous material under equal loading condition due to plastic mismatches. We also demonstrated that both different yield strength and strain hardening mismatch have an analogous effect on interfacial crack-tip constraint under small scale yielding condition via slip-line field theory. Finally it was shown that $J_1/J$ near interfacial crack tip in strain hardening bi-material is dominantly affected by the strain hardening of joining material rather than $T$-stress. The interpretation of higher constraint and higher crack driving force in the lower strength/hardening material was briefly given in view of bi-material fracture behavior. Decomposition of $J$-integral may provide a fundamental tool for predicting the fracture toughness of a bi-material from the fracture toughness of constituent materials. However, it should be also noted that the contribution to the total $J$-integral from the weaker material is strongly dependent on the distance from the crack tip. One possible explanation to this is that at material point level non-proportional stressing is experienced in the case of mismatch (this effect is stronger closer to the crack tip), and the basis for the “energy interpretation of the $J$-integral” becomes blurred. Furthermore, the similarity in stress field does not exist between the homogeneous and mismatched specimen. These aspects raise the question of whether decomposition of the $J$-integral would be a fruitful path to follow when trying to predict the fracture toughness of mismatched specimen based on fracture toughness of the constituent material. The resolution of these aspects may merit the further research works.

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References


