An area-average approach to peening residual stress under multi-impacts using a three-dimensional symmetry-cell finite element model with plastic shots

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\textbf{Abstract}
We estimate realistic peening residual stress based on area-averaged solution using a 3D multi-impact symmetry-cell finite element (FE) model. The analytical model includes elaborate factors reflecting actual peening phenomena and plastic shot effect. Area-averaged solution is much closer to X-ray diffraction (XRD) experimental solution than four-node-averaged solution in plastic shot FE model. The area-averaged solution, moreover, converges to the perfect equi-biaxial stress state. From this, based on the area-averaged solution, we obtained the FE Almen curve, and then derived related equations among FE arc height, FE coverage and shot velocity. The FE Almen curve corresponds well with experimentally obtained by Kim et al. Using the FE Almen curve, we examine the FE area-averaged solution in major peening materials. The FE solutions of surface, maximum compressive residual stress and deformation depth quite reach experimental solutions. The FE Almen curve is thus confirmed to be useful for estimation of residual stress solution. Consequently, we validated that the concept of area-averaged solution is the systematical analytical method for evaluation of real peening residual stress.

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\section{Introduction}
Shot peening has been widely used in automotive, power plant and aerospace industries to improve fatigue life of mechanical parts by generating useful compressive residual stress on the surface of metal. Peening residual stress is a main factor directly affecting the material behavior such as fatigue fracture, corrosion and wear. Therefore it is quite important to quantitatively evaluate residual stress for various peening conditions. An experimental Almen saturation curve used for prediction of the capacity of peening equipment and peening intensity is usually adopted in shot peening process. Using main parameters such as arc height, peening coverage and impact velocity which correspond to the curve, we can estimate peening residual stresses of various materials. Normally peening residual stress is measured by experimental method [3–7] using X-ray diffraction (XRD). However, the XRD measurement method is needed considerable cost, time and skillful technique. For this reason, the residual stress has been generally predicted by theoretical methods [8–10], and recently evaluated by lots of studies using finite element (FE) analysis.

In the early stage of FE analyses for peening, single impact and indentation FE models were largely used [11–16]. Recently, the single shot impact analytical model also was introduced by some researchers [17,18]. These FE analysis studies evaluated the residual stress field by assuming the single shot impact as 100% peening coverage on the surface of shot peened material. Multi-impact FE analyses closer to real shot peening phenomena have been performed in large numbers thereafter [19–22]. In these works, however, real peening phenomena including plastic deformation of shot were not sufficiently applied to the FE analyses, and examination of convergence to equi-biaxial stress and comparison between FE and experimental solutions were excluded. Kim et al. [1] have therefore proposed a 2D FE model including more realistic combined factors and plastic shot effect in a single shot impact, and extended it to a 3D FE model in multi-impacts [2]. Applying peening phenomena with FE coverage, impact sequence and cycle-repetition to the model additionally, they obtained an improved solution of equi-biaxial peening residual stress. However, previous FE analysis works merely gave single-node solutions on the surface of FE model. Namely, they did not consider concept of the area-averaged stress.
solution. Consequently, it is clear that some amount of error between the FE solution and the experimental solution exists. Because generally experimental XRD residual stress is measured at area where is irradiated by X-ray. Boo et al. [23] experimentally measured residual stress solution at the £4 mm circular area irradiated by X-ray on the surface of WC–Co hardmetal. Hong et al. [24] evaluated the XRD residual stress at the 2 mm /C2 7 mm area.

Recently Jakobsen et al. [25] estimated characteristic of strain at the 0.25 mm /C2 0.5 mm area irradiated on surface of material by 3D X-ray beam source. Especially, in the experimental peening studies, Prevey and Cammett [26] explained the relationship between area-averaged solution at the 5 mm /C2 5 mm area and peening coverage experimentally. Kirk and Hollyoak [27] obtained various area-averaged solutions of surface residual stress at the areas with 4 mm x 4 mm, 12 mm x 1 mm and 4 mm x 1 mm. For this reason, in this work, we evaluate the peening residual stress quantitatively using a new analytical approach based on concept of the area-averaged solution. Considering the experimental area-averaged solution, in this work, we first obtain the area-averaged FE solution from total nodes included at each cross-section of a 3D symmetry-cell model suggested by Kim et al. [2] in multi-impacts.

Moreover, prior studies did not consider Almen curve with arc height and peening coverage, which are essential to explain the real shot peening phenomenon. The Almen curve therefore has huge meaning in a numerical approach to quantitative evaluation of peening residual stress. Therefore, we predict the FE Almen curve using area-averaged solution, and derive the relationship among the FE arc height, FE peening coverage and impact velocity, and confirm usefulness of the FE Almen curve comparing the FE Almen curve with experimental curve. Substituting FE peening coverage and FE arc height into the derived equations, we obtain the impact velocity. Adopting the velocity to the FE model, we obtain four-node-averaged and area-averaged solutions of peening residual stresses after FE analysis, and we then compare these FE solutions with XRD experimental solution. Comparing FE solutions with XRD experimental solution, we examine closeness to experimental solution on the surface, maximum compressive residual stress and deformation depth. Ultimately, we propose the validation of the 3D multi-impact FE model integrated with FE Almen curve and plastic deformable shot based on area-averaged solution.

2. A 3D finite element model for area-averaged solution of peening residual stress

2.1. Finite element modeling and boundary conditions

Using the 3D symmetry-cell FE model proposed by Kim et al. [2] as shown in Fig. 1, we obtain area-averaged FE solution of peening residual stress in multi-impacts. We fixed S, which means a distance between shots (or a side of a cross-section of symmetry-cell), on the same shot radius R as S = R = 0.4 mm. This is because when
we consider both the symmetries of the cell and shot, $S$ should be greater than or equal to $R$ and the effect of stress interference becomes maximum at $S = R$. The symmetry-cell includes the combined factors and shot peening phenomena [1,2]. We then obtain the four-node-averaged residual stress of four impact locations (A–D) and area-averaged one of full nodes of the cell at each point along depth ($d$) direction. In this analysis, we use commercial FE analysis program ABAQUS [28]. In shot peening, to analyze large deformation of elasto-plastic material, we adopt NLGEOM (nonlinear geometry) option in ABAQUS Explicit code with 3D eight-node reduced integral elements (C3D8R; ABAQUS) [28]. As boundary conditions, we fix the bottom of the symmetry-cell completely ($U_x = U_y = U_z = 0$), and impose symmetric displacement conditions on the four sides ($U_x = 0$ or $U_z = 0$). Shots are modeled as 1/4 on the basis of A–D locations of symmetry-cell, and impose symmetric displacement conditions on the two sides ($U_x = 0$ or $U_z = 0$). We place contact surfaces on both between material and shot (contact surfaces, ABAQUS) [28] and apply penalty algorithm widely adopted for dynamic contact/impact FE analyses [29,30]. Initial shot velocity $v$ is calculated as 55 m/s using equations derived by Kim et al. [1,2] from experimental Almen curve.

2.2. Material properties and verification of finite element model

AISI4340, SAE170, AISI4140 and SPS8, which are often subject to shot peening process, were chosen in this study. Especially, SAE1070 material is adopted in Almen strip FE model which is used for FE Almen saturation curve. AISI4340 steel was tempered for 2 h in 230 °C after quenching in 815 °C. Tensile tests provided material properties as yield strength $\sigma_y = 1510$ MPa, tensile strength $\sigma_t = 1860$ MPa, elastic modulus $E = 205$ GPa, Poisson’s ratio $\nu = 0.25$, and density $\rho = 7850$ kg/m$^3$. AISI4140 steel was tempered for 2 h in 450 °C after quenching in 850 °C. As material properties, yield strength $\sigma_y = 1390$ MPa, tensile strength $\sigma_t = 1700$ MPa, elastic modulus $E = 210$ GPa, Poisson’s ratio $\nu = 0.28$, and density $\rho = 7850$ kg/m$^3$. SPS8 steel was treated by vacuum heating for inhibition of oxidization and decarbonization, and was tempered for 90 min in 420 °C after quenching in 910 °C. Yield strength $\sigma_y = 1630$ MPa, tensile strength $\sigma_t = 1920$ MPa, elastic modulus $E = 210$ GPa, Poisson’s ratio $\nu = 0.03$, and density $\rho = 7850$ kg/m$^3$. We also performed tensile tests using Almen strip of A-type. From the tensile tests we obtained the mechanical properties such as yield strength $\sigma_y = 1380$ MPa, elastic modulus $E = 210$ GPa, Poisson’s ratio $\nu = 0.25$ and density $\rho = 7850$ kg/m$^3$, and adopted them to FE model. We also selected cut wire round shot (CWRS) for FE shot model. As its mechanical properties, yield strength

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**Fig. 3.** Comparison of full-nodes averaged residual stress with four-node-averaged residual stress in (a) RS, (b) EDS and (c) PDS.

**Fig. 4.** Convergence to equi-biaxial stress in (a) 4 cycles and (b) 5 cycles.
σ_y = 1470 MPa, tensile strength σ_t = 1840 MPa, elastic modulus E = 210 GPa, Poisson’s ratio ν = 0.3, density ρ = 7850 kg/m³, diameter of shot D = 0.8 mm, and we considered rigid (RS, rigid shot), elastic (EDS, elastic deformable shot) and plastic shot (PDS, plastic deformable shot) FE models.

We adopted the minimum element size L = 0.02 mm, the material damping coefficient ζ = 0.5 and the dynamic friction coefficient μ = 0.2 which were decided in 2D single impact analysis by Kim et al. [1]. The symmetry-cell FE model, which gave the best convergence to equi-biaxial stress and approached to random impact [2], was used in this work. The height of symmetry-cell h = 1.5 mm. To obtain FE Almen saturation curves, we decided the height of FE symmetry-cell for Almen strip-A as h = 1.27 mm (SAE J442) [31].

3. Finite element analyses for node-averaged and area-averaged solutions

Fig. 2 presents the meaning of four-node- and full-node-averaged solutions at the cross-section of FE symmetry-cell based on area irradiated by X-ray. Four-node-averaged solution is obtained by impact of shots at the A-D locations on the surface of symmetry-cell, and full-node-averaged solution is obtained at all nodes forming the cross-section (0.4 mm × 0.4 mm) at each depth of the symmetry-cell. Here, the full-node-averaged solution is called the area-averaged solution. From FE analysis, we compare area-averaged solution with four-node-averaged solution with three FE shots: rigid, elastic and plastic.

Fig. 3 shows distributions of residual stress with four-node-averaged and area-averaged solutions in FE shot models. The XRD experimental solution of AISI4340 steel is based on arc height 0.36 mm A and peening coverage 200%. To obtain FE solutions corresponding to the XRD solution, we used impact velocity v = 55 m/s by substituting the parameters derived from experimental Almen saturation curve into the equations [1,2]. We compared the FE solution with XRD solution after obtaining the four-node-averaged and area-averaged solutions at the A-D locations of symmetry-cell in single cycle (1 cycle = 4 shots) and multi-cycle (4 cycle = 16 shots) impacts. In all FE shot models, the four-node-averaged FE solution greatly differs from the XRD experimental solution, while area-averaged solution quite approaches the XRD solution. In Fig. 3a and b, rigid and elastic FE shot models give the area-averaged solutions of compressive residual stresses bigger than the XRD residual stress. In single cycle impact with 100% peening coverage, the FE solutions are bigger than the XRD experimental solution, however, with more than 100% peening coverages, the difference between the FE solutions and the XRD one more increases by reason that the energy capacity of material increases in impacts. In this work, AISI4340 material has 200% peening coverage [32], and therefore rigid and elastic FE shot models with 200% peening coverage would produce large errors of residual stress comparing to the XRD experimental result. Contrary to both FE shot models, plastic shot produces excellent area-averaged solution very close to the XRD residual stress as shown in Fig. 3c. From this point, we use the plastic shot FE model in this analysis from now on. We thus confirmed that a 3D symmetry-cell with plastic...
Fig. 7. Residual stresses with various impact cycles in 2D single shot impacts.

Fig. 8. Four-node-averaged residual stresses with various impact cycles in 3D multi-impacts.
shot is a quite optimal FE model for area-averaged solution of peening residual stress in multi-impacts.

Fig. 4 shows area-averaged solutions with plastic shot FE model converge to perfect equi-biaxial stress in multi-impacts with 4 and 5 cycle-repetitions. Previous FE analysis works, the concept of area-averaged solution has never been examined. Especially, it should be noted that the area-averaged solution exceedingly reproduces experimental measurement by XRD, which is very realistic. The FE model will be used for derivation of FE Almen saturation curve in the next section.

4. Finite element Almen saturation curve based on area-averaged solution

Generally, Almen curves are used as a standard to evaluate the efficiency of peening machine, and important data to predict the peening intensity. Using the curve, we can easily choose and control the parameters corresponding to Almen intensity. Guagliano [19] explained the relationship between impact velocity and arc height substituting the FE residual stress obtained by 3D multi-impact analyses as

\[ H = \frac{3Mi^2}{2Ebh} \]  
\[ M = \int_A \sigma_x(y)ydA \]

where \( H \) is the arc height which means the curved height of the bended strip with the gauge length \( l \) of the Almen gauge, \( M \) is the moment induced by residual stress, \( E \) is the elastic modulus of Almen strip, \( b \) and \( h \) are the width and thickness of the strip. Guagliano's FE arc height \( H \) was, however, calculated at a single-node only, so single-node solutions were used instead of area-averaged solutions. In addition, he did not considered shot peening coverage which is an important parameter, and used rigid shot FE model.

A 3D symmetry-cell FE model was established to obtain arc height of the A-type Almen strip as shown in Fig. 5. Here, the length of Almen strip \( L \) is 76 mm, and the gauge length \( l \) is 31.75 mm (SAE J442) [31]. The width of the strip-A \( b \) is 19 mm, the thickness of the strip-A before shot peening \( h \) is 1.27 mm. \( h_a \) is the averaged height of symmetry-cell (=the thickness of Almen strip-A) considered dents formed by impact of shots after peened (in Fig. 5). The \( y \) is the distance from neutral axis of Almen strip-A to the surface or bottom face, \( \sigma_x(y) \) is the value residual stress magnitude of \( x \)-direction at the \( y \) distance.

In this study, we obtained the FE Almen curves based on area-averaged solutions using the above Eqs. (1) and (2), and derived the relationship among the FE arc height, FE peening coverage and impact velocity. For the calculation of FE arc height, Guagliano [19] made an error that \( L = 76 \) mm (=\( L \)) was substituted for \( l \) in Eq. (1), while we used \( l = 31.75 \) mm obeying the SAE J442 standard [31]. Guagliano [19] also substituted \( h = 1.27 \) mm for the thickness of Almen strip-A because he did not consider variation of thickness due to shot peening. However, considering the variation of thickness, we substituted \( h_a \) for \( h \).

The distribution of residual stress generated by multi-impact analysis in the inner parts of the Almen strip-A FE model is presented in Fig. 6 [19]. Fig. 6a shows the peening residual stress and reaction force immanent in Almen strip-A FE model when

![Fig. 9. Full-node-averaged residual stresses with various impact cycles in 3D multi-impacts.](image-url)
the bolts are fixed, and Fig. 6b shows the state of stress balance of the bended strip-A model after the bolts are removed. Using the distribution of peening residual stress shown in Fig. 6a, we calculated the moment of the strip-A, and then estimated the FE arc height substituting the moment into Eq. (1). We included then the effect of FE peening coverage on the FE Almen curve. In the 2D FE analysis, we set the peening coverage $C$ generated by a single shot impact to be 100%. Then $C$ is 200% for 2 shot (=2 cycles) impacts, and $C$ is 300% for 3 shot (=3 cycles) impacts. In the 3D multi-impact FE analysis, we set peening coverage $C$ generated by a single cycle impact which consists of four shots to be 100%. Then $C$ is 200% for 2-cycle impacts by 8 shots, and $C$ is 300% for 3-cycle impacts by 12 shots. From this, we can obtain the equations derived from the relationship between calculated FE arc height and peening coverage. Durability parts used in major industries give various peening coverages more than 100%, and they are generally in the range of 100–300% [33–36]. We thus set the range as 100–300% in this work.

Fig. 7 shows the distribution of FE peening residual stress when impact velocity limit is from 40 to 70 m/s in the 2D single shot impact. Figs. 8 and 9 illustrate the distribution of FE peening residual stresses applying the same velocity limit to the 3D FE model after multi-impacts with area-averaged and four-node-averaged solutions, respectively. We calculate the FE arc heights substituting the solutions of peening residual stress shown in Figs. 8 and 9 into Eqs. (1) and (2).

Fig. 10 shows the relationship between FE peening coverage $C$ and arc height $H$. The FE $C$–$H$ curves are based on a single-node solution in 2D FE analysis, four-node-averaged and area-averaged FE solutions in 3D FE analyses. We compared then the FE curves with experimental curve. The FE arc heights of the 2D single-node and 3D four-node FE $C$–$H$ curves are quite higher than experimental arc heights. The FE arc heights of 3D area-averaged FE $C$–$H$ curves are much close to experimental one.

We also obtained various FE Almen curves as shown in Fig. 11a–c. Fig. 11a presents the relationship between the number of shot $N_{\text{shot}}$ and FE arc height $H$. It is greatly similar to experimental Almen curve obtained by Kim et al. [1,2], and can be expressed as

$$H^v = A(1 - e^{-0.5N_{\text{shot}}})$$  \hspace{1cm} (3)

We considered limit of the $N_{\text{shot}}$ and $v$ as $1 \leq N_{\text{shot}} \leq 12$ and $30 \leq v \leq 80$, respectively. Table 1 shows the numerical values of variables and coefficients for Eq. (3). Fig. 11b shows variation of FE height $H$ with FE peening coverage $C$. The FE arc height $H$ can therefore be expressed as a function of $C$ as follows:

$$H^v = B_1C^2 + B_2C + B_3$$ \hspace{1cm} (4)

The variables and coefficients of Eq. (4) are given in Table 2. Fig. 11c shows variation of impact velocities $v$ with various arc heights $H$. It is expressed by a linear function of arc height as

$$v^C = C_1H - C_2$$ \hspace{1cm} (5)

As peening coverage increases, the slope slightly decreases and eventually converges to a certain value. Table 3 shows the numerical values of variables and coefficients (Eq. (5)).

Like this, we can get the impact velocities for input using Eqs. (3)–(5) derived by the FE Almen curves. The equations are used
5. Experimental verification of FE area-averaged solution in various main peening materials

We can more quantitatively evaluate peening residual stress using the FE Almen curve derived in the prior section as well as maximize efficiency of peening process by prediction of compressive residual stresses generated on the surface of materials. In this paragraph, using the FE Almen curve, we determine area-averaged FE solutions in various main materials used for shot peening and compare them with XRD solutions.

Fig. 12 shows distributions of area-averaged FE solution and XRD experimental solution in AISI4340 steel which is usually used for aerospace landing gear. AISI4340 steel gives the arc height $H_{\text{A}} = 0.36 \text{ mm A}$ and peening coverage $C = 200\%$ [32]. In Table 3, when $C$ is 200%, the coefficients $C_1$ and $C_2$ are 175.0 and 9.4, respectively. Substituting $C_1 = 175.0$, $C_2 = 9.4$ and the arc height $H = 0.36 \text{ mm A}$ into Eq.(5), we obtain the impact velocity $v$ as $54 \text{ m/s}$. Adopting the velocity to this analysis, we obtain four-node-averaged and area-averaged solutions of peening residual stresses, and we then compare the FE solutions with XRD experimental solution. Especially, area-averaged solution is greatly close

![Table 1](image)

![Table 2](image)

![Table 3](image)
which means $H = 0.26\text{ mm A}$ and peening coverage or general machine parts. AISI4140 steel gives the arc height $XRD$ experimental solution in AISI4140 steel used for automobile Comparison of finite element solutions with XRD solution in AISI4140 steel.

Fig. 13 shows the best distribution of the area-averaged solutions which different than experimental one. However, overall, FE solutions show the best convergence to the surface, maximum compressive residual stress lies a bit to experimental solution on the surface. When deformation depth $d$ is 0.08 mm, FE maximum compressive residual stress lies a bit different than experimental one. However, overall, FE solutions approach experimental solution.

Fig. 13 shows distributions of area-averaged FE solution and XRD experimental solution in AISI4140 steel used for automobile or general machine parts. AISI4140 steel gives the arc height $H = 0.26\text{ mm A}$ and peening coverage $C = 100\%$ [37]. In Table 3, when $C$ is 200%, the coefficients $C_1$ and $C_2$ are 193.0 and 10.5, respectively. Substituting $C_1 = 193.0$, $C_2 = 10.5$ and the arc height $H = 0.26\text{ mm A}$ into Eq. (5), we obtain the impact velocity $v$ as 39.6 m/s. Adopting the velocity to FE model, we obtain four-node-averaged and area-averaged solutions of peening residual stresses after FE analysis, and we then compare the FE solutions with XRD experimental solution. In AISI4140 steel, area-averaged FE solution was close to XRD experimental solution than four-node-averaged solution. Especially, area-averaged solutions give the best convergence to the surface, maximum compressive residual stresses and deformation depth.

Fig. 14 shows the distributions of area-averaged FE solution and XRD experimental solution in SPS8 steel often used for automobile spring. AISI4140 steel gives the arc height $H = 0.37\text{ mm A}$ and peening coverage $C = 85\%$ [38]. Generally when the range of peening coverage is $85 \leq C \leq 98\%$, it is simply called as full-coverage which means $C = 100\%$. For this reason, in this analysis, we apply $C = 100\%$ to the FE model. In Table 3, when $C$ is 100%, the coefficients $C_1$ and $C_2$ are 193.0 and 10.5, respectively. Substituting $C_1 = 193.0$, $C_2 = 10.5$ and the arc height $H = 0.37\text{ mm A}$ into Eq. (5), we obtain the impact velocity $v$ as 61 m/s. In the material, area-averaged FE solution is quite closer to XRD experimental solution than four-node-averaged solution.

As above, with comparing the FE solution with experimental solution in various main peening materials, we therefore validated the FE analytical method using FE Almen saturation curve based on area-averaged solution.

6. Conclusions

Residual stress due to shot peening, measured by XRD experimental method, gives area-averaged solution at the area irradiated by X-ray on the surface of parts. Previous analysis works merely gave FE solutions only at a single-node on the surface of FE model after impact of shots, and were never considered the area-averaged solution. Therefore it is clear that an error between FE solution and experimental solution is large. For this reason, in this study, using the 3D multi-impact symmetry-cell [2], we obtained FE area-averaged solution for peening residual stress. Symmetry-cell includes combined factors and plastic shot, and applied peening phenomena to the model enough. We then compared the FE solutions with experimental solution. In plastic FE shot model, area-averaged solution greatly closed to XRD experimental solution than four-node-averaged solution. The area-averaged solution, moreover, converges to the perfect equi-biaxial stress state. From this, we derived related equations among FE arc height, FE peening coverage and impact velocity using the FE Almen saturation curves based on area-averaged solution. FE Almen curves corresponded well with those experimentally obtained by Kim et al. [1,2]. From this, we derived three equations of the relationship among the FE arc height, FE peening coverage and shot velocity. Using the equations, we obtained FE area-averaged solutions for three materials such as AISI4340, AISI4140 and SPS8 often used in peening process, and then compared their FE solutions with XRD experimental one. In all materials, FE solutions of the surface, maximum compressive residual stress and deformation depth were very close to experimental solution. We thus confirmed validity of FE Almen saturation curve for prediction of peening residual stress in various peening materials and mechanical parts. As mentioned above, we validated that the concept of area-averaged solution is the systematical analysis method for evaluation of real peening residual stress ultimately, since the FE solution exceedingly represents the XRD experimental solution.
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