Failure Assessment Diagrams of Semi-elliptical Surface Crack with Constraint Effect

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Abstract. For accurate failure assessment, a second parameter like T-stress describing the constraint is needed in addition to the single parameter J-integral. In this work, selecting the structures of surface-cracked plate and pipe, we perform line-spring finite element modeling, and accompanying elastic-plastic finite element analyses. We then present a framework, which includes the constraint effects in the R6 FAD approach for failure assessment of cracked-structures.

Introduction

Due to their intrinsic 3D nature, surface crack problems are generally approached by numerical technique. As a semi-3D way of surface crack analysis, to overcome the complexities of surface crack, there is a model called the line-spring contrived by Rice and Levy [1]. Traditionally, crack-tip fields are described by J-integral of elastic-plastic fracture mechanics. However, the validity of these single parameters are limited by geometric configuration or loading conditions of the specimen. Betegon and Hancock [2] demonstrated that elastic-plastic crack-tip stress fields can be described by the first two terms (J-T) of Williams eigen expansion.

On this background, we perform the elastic-plastic FE analyses for semi-elliptical surface cracks in plates and pipes with line-spring model. We then include the constraint (T-stress) effects in the R6 FAD approach to the integrity assessment of cracked plates and pipes.

A Brief on Line-spring Model

Line-spring model was contrived by Rice and Levy [1] to effectively calculate the stress intensity factor along a part-through surface crack in a plate/shell structure. Parks and co-workers [3] extended the concept to elastic-plastic crack analyses with the incremental theory of plasticity.

The virtue of line-spring model lies in simplification of a complex 3-D problem into a tractable 2-D one. There exists additional compliance in a plate/shell due to the presence of a part-through surface crack, when compared with the non-cracked case. The main feature of line-spring model is that this additional compliance is considered by a set of spring (Fig. 1). Consider a surface crack of length 2\(c\) in a plate of thickness \(t\). Here coordinate \(x\) is the distance from the center \((x = 0)\) of semi-elliptical surface crack, and \(a(x)\) is the crack depth at position \(x\). The compliance of line-spring finite element in Fig. 1 changes with the crack depth \(a(x)\). If the crack length 2\(c\) is much greater than the thickness \(t\), the surface crack becomes nearly a plane strain SEC (Single Edge Crack) specimen. In view of this limiting case, the additional compliance at position \(x\) is taken as that of the plane strain SEC specimen with crack depth \(a(x)\) and thickness \(t\) [1,3]. When combined plate/shell and line-spring model are analyzed, the force \(N(x)\) and moment \(M(x)\) in line-spring finite element are obtained. The fracture parameter \(J(x)\) are then given as those of plane strain SEC specimen with crack depth \(a(x)\) and thickness \(t\) subject to combined \(N(x)\) and \(M(x)\).

By comparing J-T values from the line-spring FE to the complete 3D FE solutions using ABAQUS software for tensile and bending loadings of a plate and a pipe (Fig. 2), we confirmed that the line-spring J-T solutions are quite acceptable in their accuracy.
Failure Assessment Diagram with Constraint Effects

**R6 failure assessment diagram.** From the Dugdale yield strip model, Harrison, Loosemore and Milne [4] presented a failure assessment curve (R6 curve) in the following form.

\[
\frac{K_I}{K_{IC}} = \frac{\sigma_f}{\sigma_c} \left(8/\pi^2\right) \ln \left[\sec\left(\frac{\pi \sigma_f}{2 \sigma_c}\right)\right]^{0.5}
\]  

(1)

Here \( K_I \) is mode I stress intensity factor, \( K_{IC} \) is fracture toughness and \( \sigma_f \) is failure stress. Plastic collapse stress \( \sigma_c \) is determined by limit analysis. R6 FAD (Failure Assessment Diagram) is an interpolation of the elastic fracture and plastic collapse presented in the \( K_r-S_r \) plane. Here \( S_r = \sigma/\sigma_c \), \( K_r = K_I/K_{IC} \), and they are proportional to the applied load via \( \sigma \) and \( K_I \). The failure assessment curve (FAC) is the trace of points \([K_r], (S_r)\) that satisfy Eq. (1). The cracked body is safe if the assessment point is inside FAC; it fails if outside the FAC.

**Fracture and plastic collapse variables.** The fracture variable \( K_r \) used in forming FAD is defined as \( K_r = \sqrt{J/J_{IC}} \) by the ratio of \( J \) from FE analysis to fracture toughness. Further, the plastic collapse load variable \( S_r \) is defined using the yield surfaces for a plane strain SEC specimen under combined tension and bending [3]. To define \( K_r \) including \( T \)-stress measuring constraint of a cracked structure, a new definition of constraint-dependent toughness \( J_{CM} \) is needed. MacLennan and Hancock [5] demonstrated that modified toughness \( J_{CM} \) varies with \( T \)-stress as

\[
\tau > 0: J_{CM} / J_{IC} = 1 \quad \tau < 0: J_{CM} (\tau) / J_{IC} = 1 - 6.7\tau - 13.8\tau^2 - 37.1\tau^3 - 11\tau^4
\]  

(2)

Here \( \tau = T/\sigma_c \). Accordingly, for cracked structure under negative \( T \)-stress loading, the fracture variable \( K_r \) takes a smaller value due to toughness increase, compared with the one without \( T \)-effect.

**Failure assessment diagrams.** Elastic-plastic line-spring finite analyses produced the FADs for a plate/pipe with a semi-elliptical surface crack. The material ASTM A710 [6] is considered. The
elastic modulus $E = 208.4 \text{ GPa}$ and Poisson’s ratio $\nu = 0.256$. Yield and tensile strength $\sigma_y = 470 \text{ MPa}$, $\sigma_t = 677 \text{ MPa}$, average flow strength $\sigma_o = 573.5 \text{ MPa}$ and hardening exponent $n = 10$. The fracture toughness $J_{IC} = 120\text{MPa-mm}$. The applied load was set as 0.6 times of the yield load of uncracked body in producing FADs. Flow strength $\sigma_t$ was used instead of yield strength $\sigma_y$ in defining the R6 curve. The dimensions of plate and crack in FE analyses are $h/b = 4, b/t = 24; c/t = 6, a/t = 0.25, 0.5$. For the case of pipe, $h/R = 10, R/t = 10; c/t = 6, a/t = 0.25, 0.5$ (See Fig. 2).

Figure 3(a) shows the $(K_r, S_r)$ values along the semi-elliptical surface crack front for the plate under remote tension. The safety factor decreases with increasing crack depth $a/t$ and gets its minimum at the crack center. Moreover, $T$-stress is negative throughout the crack front in tensile loading. Thus $K_r$ values of load state points along the crack front decrease about 60% for $a/t = 0.25$ and 0.5. This comes from the increase of toughness $J_{CM}$ due to negative $T$-stress. Figure 3(b) shows the values of $(K_r, S_r)$ for the plate under remote tension.

Figure 4(a) shows the $(K_r, S_r)$ values for the pipe under remote tension. In this case, $S_r$ spreads more widely along the crack front than the plate case. When negative $T$-stress is considered, $K_r$ values of load state points along the crack front decrease about 75% for $a/t = 0.25$ and 0.5. Figure 4(b) shows the $(K_r, S_r)$ values for the pipe under remote bending. All the $(K_r, S_r)$ points of load state locate outside the R6 curve for $a/t = 0.5$. Some $(K_r, S_r)$ points near crack center also locate outside the R6 curve for $a/t = 0.25$. When negative $T$-stress is considered, $K_r$ values of load state points along the crack front decrease about 77% for $a/t = 0.25$, and for $a/t = 0.5$ the safety factors become greater than 1 for $x/c > 0.3$. This also comes from the increase of toughness $J_{CM}$ due to negative $T$-stress.

![Fig. 3 Modified and unmodified FAD of surface cracked plate under remote tension, bending for two crack depths.](image1)

![Fig. 4 Modified and unmodified FAD of surface cracked pipes under remote tension, bending for two crack depths.](image2)
Failure Assessment Curves with Constraint Effects

It is desirable to derive a more realistic Failure Assessment Curve (FAC) for accurate assessment of hardening ASTM A710 material. Consider a straight-line $y_1 = \left(J_e/J_{IC}\right)^{1/2}$ and a curved-line $y_2 = \left(J_e/J_{US}\right)^{1/2}$, where $J_e$ is elastic $J$. With increasing load, the crack start to grow since $J_{US} = J_{IC}$ at $y_1 = y_2$. The curve $y_2$ take the various functional forms according to the structural configurations [5]. With these concepts, we produce FACs based on the $J$-values at the surface crack center.

Figure 5 presents the three failure assessment curves: R6 curve, FACs with/without $T$-stress effect for (a) plate and (b) pipe. The FACs were produced by applying a rotation of 20 times of yield rotation at the ends of line-spring model. The $(K_s, S_r)$ are those for applied bending loads set as 0.6 times of the yield loads of uncracked bodies. Here $(S_r)$ for FACs and $S_r$ for FAD is again defined with the yield surfaces of SEC specimen [3]. Figure 5(a) demonstrates that $T$-stress is positive along the crack front for $a/t = 0.5$ in the plate under bending; thus the FACs with/without $T$-effect are identical. On the other hand, $T$-stress is negative along the crack front for $a/t = 0.25$ in the plate under bending. The FAC with $T$-effect thus lies above that without $T$-stress effect due to increased $(K_s)$. Figure 5(b) demonstrates that $T$-stress is negative throughout the crack front for $a/t = 0.2$ and 0.5 in the pipe under bending in contrast to the plate case. Consequently, the $(K_s, S_r)$ points for $a/t = 0.5$ locate outside the FAC without $T$-stress, but inside the FACs with $T$-effect except crack center point (A).

Fig. 5 Modified and unmodified FAC and FAD of surface cracked (a) plate (b) pipe under remote bending for two crack depths.

References


