An Indentation Method Based on FEA for Equi-biaxial Residual Stress Evaluation

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Keywords: Equi-biaxial residual stress, conical indenter, Young’s modulus, Yield strength, Strain-hardening exponent, contact diameter

Abstract. An indentation method to determine equi-biaxial residual stress is proposed by examining the data from the incremental plasticity theory based FE analyses. We found that hardness is strongly dependent of the magnitude and sign of residual stress and material properties. We then selected some normalized parameters minimally affected by material properties and tip radius. With numerical regressions of the data obtained, we proposed new formulae for residual stress evaluation. The new approach provides a substantial enhancement in accuracy compared with the prior methods.

Introduction

Residual stresses are formed by diverse processes: surface coatings, welding, machine working, heat treatments, and shot peening [1]. The residual stresses in materials affect the behavior of material including fatigue, fracture, corrosion, abrasion, and friction. For this reason, various experimental measuring techniques have been developed such as X-ray and neutron diffraction, strain/curvature measurements, beam bending, hole drilling, layer removal, chemical etching [1-2]. Each of these methods, however, has a shortcoming in the light of accuracy, sensitivity, resolution, cost, specimen preparation, material types, and geometry of structure. An indentation test is used to measure material properties [3,4]. It is nondestructive and easy to use, and moreover it can be applied to small specimen and parts in present structural use. For this reason, the indentation test has many advantages to evaluate residual stress. To evaluate residual stresses using indenter, it is necessary to secure the indentation data at non-residual state.

In the first stage, attention was attracted to the variation of hardness with the direction and magnitude of residual stresses. Tsui et al. [5] and Bolshakov et al. [6] investigated the effect of residual stresses on hardness, contact area and Young’s modulus using experimental work and FEA. They showed that residual stress is not related material hardness, but closely to the pile-up of material. On the assumption that triaxial stress does not affect material hardness, Suresh and Giannakopoulos [1] proposed a method evaluating equi-biaxial residual stress and plastic strain under frictionless condition. Lee and Kwon [7] modified the method of Suresh and Giannakopoulos. They decomposed equi-biaxial residual stress into two stresses, namely, mean stress and deviatoric stress. They assumed that only the component of deviatoric stress, which is parallel to the direction of indentation, is related to plastic deformation.

The above-mentioned studies, however, took no notice of the effects of material properties and the radius of indenter tip on residual stress. The indenter tip-rounding is realistically inevitable, which naturally affects the measured depth of indentation. Thus the prior studies on the evaluation of residual stress are not yet fully validated for all kind of materials and indenters with inevitable tip-rounding. In the present work, we would like to present a novel method for evaluating elastic/plastic equi-biaxial residual stress based on FE solutions using the incremental theory of plasticity. On the basis of the preliminary study, we analyzed the effects of friction coefficient, indenter tip radius on the shape of indentation load-depth curves using incremental plasticity FE solutions. In the present work, we select some dimensionless indentation parameters which minimize the effects of material properties and tip-rounding. With numerical regressions of the data obtained, we finally propose new indentation formulae for residual stress evaluation.
Prior Indentation Theories for Residual Stress Evaluation

Suresh and Giannakopoulos [1] suggested a methodology to determine surface residual stresses using sharp indentation invoking the invariance of contact pressure. For tensile and compressive residual stresses, they assumed the following relation.

\[ P_a = P + \sigma_R f A \]  

(1)

where \(A\) is projected contact area at unloaded state. Geometric factor \(f=1\) for tensile residual stress and \(f=\sin\alpha\) for compressive residual stress, and \(\alpha\) is the indenter tip-angle as given in Fig. 2. \(P\) and \(P_o\) are the maximum load with and without residual stress, respectively (Figs. 1-2). Suresh and Giannakopoulos (SG) [1] explained the magnitude of virtual load due to residual stress should change because virtual and indentation loads act counter to each other under compressive residual stress. For Vickers indenter, the geometric factor of SG is around 0.375, and that of Lee and Kwon [7] is around 0.667. On the other hand, Atar et al. [8] illustrated that the geometric factor in ceramic thin films should be unit by comparing residual stresses obtained from indentation test with those determined by X-ray diffraction for compressive residual stress. We note that it is not based on any physical reason to distinguish between tensile and compressive residual stresses.

Residual Stress Evaluation Based on FEA

FE Modeling of Indentation Test. Figure 3 shows an FE model of the sharp indentation test. We perform nonlinear geometry change FE analyses using isotropic elastic-plastic material, which obeys \(J_2\) flow theory. Considering both loading and geometric symmetries, we use the four-node axisymmetric element CAX4 [9]. Our preliminary analyses revealed that the eight-node CAX8 element in ABAQUS [9] has a trouble of discontinuous equivalent plastic strain at its mid-node. The lower degree of CAX4 shape function is supplemented by placing fine elements with size 0.0625\% of indenter diameter at the material contact surface. Multi-Point Constraints [9] is conveniently used at the transition region where element size changes. But constrained mid-nodes of MPC tend to give discrete stress and strain values. We thus adopt trapezoidal elements in the transition region near the contact surface, and use MPC in the transition region far from the contact surface. FE model consists of about 1800 elements. We also place contact surfaces [9] at both material and indenter surfaces of Fig. 3. Axisymmetric boundary conditions are imposed on the nodes on the axisymmetric axis. The rigid indenter with round-tip moves down to penetrate the material with the bottom of the specimen fixed.

![Fig. 1 Load-depth curves for various residual stresses.](image1)

![Fig. 2 Schematic of sharp indentation profiles considering tip rounding effect.](image2)
To preset the state of equi-biaxial residual stress in FE model, we studied two kinds of methods. One is initial boundary condition method by imposing initial prescribed radial displacements at the outer boundary of FE model, the other is initial stress method by using ABAQUS option [9]. To use initial stress option, radial displacement at outer boundary must be fixed as zero value. Comparison of two methods has led us to confirm that they produce almost identical results. In the present work, we use initial stress option since it is easier than boundary condition method in imposing residual stresses on FE mesh.

**Some Remarks of Prior Indentation Theories.** Before investigating residual stresses using indentation test based on FEA, we examined the basic relationship between residual stress $\sigma_R$ and hardness $H (= P/A)$. Figure 4 shows the variation of hardness with indentation depth for a given residual stress, indicating that hardness is independent of indentation depth.
Figure 5 shows the variation of hardness with residual stress for various material properties. Here $H_f$ is hardness under residual stress free condition. It is clarified in Fig. 5 that hardness is strongly dependent of the magnitude and sign of residual stress and material properties, while prior indentation studies [1,5,6] asserted that hardness is insignificantly affected by the residual stress. Prior studies were limited to narrow range of material with little thought of various materials. Consequently, significant errors in predicted residual stress are unavoidable when formulae based on hardness invariance are used.

On the assumption that triaxial stress does not affect material hardness, Suresh and Giannakopoulos [1] proposed a method evaluating equi-biaxial residual stress. Their basic Eq. (1) for $f = 1$ can be written as

$$\sigma_R = \frac{P_o - P}{A}$$  \hspace{1cm} (3)

The values computed with the above expression are plotted in Fig. 6. It is revealed from this figure that predicted residual stress virtually depends on material properties. Especially, the error of predicted residual stress increases with strain-hardening exponent. For compressive residual stress, Suresh and Giannakopoulos calculated residual stress by Eq. (1) for $f = \sin \alpha$, but it does not reduce the scattering of Fig. 6, either. If strain-hardening exponent $n$ is 3 for compressive residual stress in Fig. 6 (open symbols in ellipse), the geometric factor $f$ is rather 1 than $\sin \alpha$.
New Numerical Approaches to Evaluate Residual Stress. Figure 7 demonstrates that the ratio between actual contact area $A$ and ideal contact area $A_t$ without considering pile-up/sink-in is an important parameter to evaluate residual stress by indentation. Pile-up/sink-in phenomenon affects contact area, mean indentation depth and constraint around subindenter, and this effect occurs consistently for all kinds of materials except for yield strain $\varepsilon_o = 0.01$ and strain-hardening exponent $n = 3$ in Fig. 7. In indentation, elastic deformation is independent of residual stress, so elastic recovery is another main parameter for the material containing large elastic strain for plastic strain at same total strain. Figure 8 reveals that the parameter considering the ratio between plastic indentation depth $h_p$ and indentation depth $h_t$ has a consistent trend for all materials. From the third order polynomial regression of Fig. 8, we express residual stress in the form

$$\sigma_R = \frac{(P_o - P) h_t}{A h_p f(A/A_t)}$$  \hspace{1cm} (4)

Equation (4) provides residual stress with an average and maximum error of less than 5% and 25% respectively for all kinds of metal.

In a case where material properties can be known or obtained from other test like a spherical indentation [3], we can estimate residual stress in a rather simple way as the pile-up/sink-in could be included in the effect of material properties. From the Kick’s law $P = Ch_t^2$ [11] for self-similar indenter, residual stress is given by the following equation.

Fig. 10 Comparison of residual stress vs. indentation parameter for (a) $\varepsilon_o = 0.001$, (b) 0.002, (c) 0.004 and (d) 0.01.
where $C$ is material constant and it can be obtained from the regression of load-depth curve. The FE solutions of Eq. (5) can be expressed with the following polynomial formula.

$$\frac{\sigma_R}{\sigma_o} = f_c \left( \frac{C_o - C}{\sigma_o}, \varepsilon_o, n \right)^j$$

with:

$$f_c \left( \varepsilon_o, n \right)^j = \alpha_0 \left( \varepsilon_o \right)^{n-j} ; \quad i = 1, 2, \quad j = 0, 1, 2, 3 ; \quad \beta_{ijk} \sigma_o^k ; \quad k = 0, 1, 2$$

Figure 10 shows predicted and real residual stresses for various material properties. Eq. (6) provides residual stress with an average error of less than 3% for all kinds of metal. This approach is truly convenient when material properties can be readily obtained, because it is free from measuring actual contact area.

**Summary**

From FE analyses of sharp indentation test, we investigated the relationships between indentation parameters and residual stresses. We found that hardness is strongly dependent of the magnitude and sign of residual stress and material properties, while prior indentation studies asserted that hardness is insignificantly affected by the residual stress. We then selected some dimensionless indentation parameters which minimize the effects of material properties and tip-rounding. With numerical regressions of the data obtained, we finally proposed new indentation formulae for residual stress evaluation. The proposed indentation approach provides a substantial enhancement in accuracy compared with the prior methods.

**References**

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10.4028/www.scientific.net/KEM.326-328.481