Overload failure curve and fatigue behavior of spot-welded specimens

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Received 12 September 2003; received in revised form 11 May 2004; accepted 21 July 2004

Abstract

The mechanical behavior of a spot-welded specimen is generally approached in angles of overload and fatigue failures. The primary issue in an overload failure is to establish an overload failure criterion. Fatigue failure of spot-welded specimens can be dealt with a fracture parameter, since a spot-weld forms a singular geometry of external crack type. In this work, we express the limit loads in terms of base metal yield strength and specimen geometries. We then present a master overload failure curve for a single spot-welded specimen in a mixed-mode load domain. The coordinates of the domain are normalized by the limit loads of single spot-welded specimens. Recasting the load vs. fatigue life relations experimentally obtained, we attempt to predict the fatigue life of various spot-weld specimens with a single parameter denoting the equivalent stress intensity factor. This crack driving parameter is demonstrated to successfully describe the effects of specimen geometry and load type in an inclusive manner. The suggested fatigue life formula for a single spot-weld can be used in the assessment of spot-welded panel structures as the fatigue strength of multi-spots is eventually determined by that of each single spot-weld.

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Keywords: Spot-weld; Limit load; Overload failure; Fatigue life; Equivalent stress intensity factor

1. Introduction

Spot-welding is widely used as a joining method for the body and main shell-type components of automobiles, such as T-type and #-type complex subframes [1] and structural rolled steel in railroad vehicles. It is self-evident that understanding the mechanical behavior of spot-welds as well as plates itself is important in structural integrity assessment and crashworthiness test. Especially, location and number of spot-weld
points and the quality of the spot-weld are primary factors, which determine the performance of spot-welded structures. Difficulties in describing the spot-weld behavior are twofold. First, from the material point of view, the spot-welded zone is non-homogeneous, since the plastic properties of the material in the (resistance) heat-affected zone (HAZ) vary from point to point. That is, the whole material properties are not simply described by the properties of the base metal. The material property distribution with the location in the HAZ should thus be carefully measured. We note here that generalized data on the material properties are too difficult to obtain. This is because the distribution of material properties in the HAZ shows a strong dependence on the welding condition, while the welding condition itself is in turn determined by the material properties and thickness of the base metal. Secondly, from the geometrical point of view, the spot-weld forms an external crack, where the spot-weld plays the role of a ligament. This geometric singularity would bring about stress concentration and accompanying excessive local deformation. Consequently fracture mechanics approach in a strict sense is required for a systematic description of spot-weld behavior. In this case, the spot-weld boundary is subject to combined tension, bending and shear; therefore, the problem becomes an intrinsically three-dimensional one. This causes a substantial difficulty in calculating the overload for instantaneous failure and the fracture parameter along the (spot-weld peripheral) crack front for gradual fatigue failure [2].

Failure analyses of spot-welds are largely divided into two: the (quasi-static) overload failure analysis and the fatigue analysis. The primary aim of overload analysis is to provide a failure curve, which facilitates the macroscopic finite element (FE) analyses of car crashworthiness [3,4]. The failure curve is usually presented in a mixed mode force domain [5] in terms of material property and some geometrical quantities such as the nugget diameter and the panel thickness. Fatigue analysis of spot-welded specimens may demand a lot of time and effort in collecting the fatigue data. Once the test data are obtained, however, the fatigue failure can be predicted by a fracture parameter describing the effects of loading type and specimen geometry in an inclusive manner.

The fracture mechanics approach for fatigue analysis of spot-welded specimens was first attempted by Pook [6] with an external crack shape of spot-welds in mind. Pook showed that experimental data from tensile-shear specimens (Fig. 1) become much less scattered, and fatigue limit is more clearly defined by reconstructing the load–fatigue life relation to stress intensity factor-fatigue life relation. The effectiveness of Pook’s method, however, is limited to the small scale yielding (SSY) condition, since only opening-mode stress intensity factor $K_I$ was taken as a crack driving parameter in spite of mixed-mode loading in tensile-shear specimens. If the load magnitude exceeds the SSY condition, fatigue failure of spot-weld specimen is governed by plastic strain, and consequently an elastic–plastic fracture parameter should be considered. In order to overcome this limitation, Wang and Ewing [7,8] selected an elastic–plastic fracture parameter, $J$-integral, as fatigue crack growth driving parameter, and analyzed fatigue test data obtained from various spot-weld specimens. Wang and Ewing described the effects of the material, shape and radius of the nugget, width and thickness of specimens, in a rather comprehensive manner, using a single parameter $J$. But as their fatigue life prediction of spot-welds depends on the loading mode whether it is tensile-shear mode [7] or pure shear mode [8], their work lacks generality and practical use. Considering that fatigue failure of overload spot-welding is caused by plastic strain in nuggets, Ono et al. [9] described fatigue life with strain measured by a strain gauge in the outer and inner surfaces of tensile-shear spot-welded specimens. But the plastic strain on the inner surface of specimens could be measured only by making a hole on the specimens; thus, the measured strain has an error due to stress redistribution.

While spot-welding is generally used in the form of multi-spots [10,11], the fatigue strength of a multi-spot-welded structure is eventually determined by the fatigue strength of each single spot-weld. Therefore, examining fatigue strength of various types of single spot-welded specimens is essential for the assessment and design of spot-welded panel structures. Moreover, in predicting the fatigue life of spot-welded specimens with a parameter $J$ as in the studies of Lee and Choi [2] and Wang and Ewing [7,8], the data of load–fatigue life should be essentially obtained in advance through various tests of spot-welded specimens.
On this background, we find the limit loads from quasi-static load–displacement curves of four types of single-spot-welded specimen as shown in Fig. 1 (coach-peel: CP, cross-tension: CT, pure-shear: PS, tensile-shear: TS). The effects of geometric parameters and the loading type on the limit load are investigated. Along with that, plastic property distributions in fusion and heat-affected zones are deduced from the measured hardness of a spot-weld section. These data on plastic property distribution turn into useful information in other FE studies such as those of Lee and Choi [2,10]. We then suggest a master failure curve of a single spot-weld. The failure curve is naturally given in a mixed mode domain normalized with limit loads. Note that heat-affectation induced hardening of the spot-welded part is the main cause of fatigue crack growth in the panel thickness direction, not in the nugget interfacial direction. We briefly discuss crack kinking. Finally the load–fatigue life data of this study are recast in terms of equivalent stress intensity factor $K_e$ where mode mixity is properly considered. The $K_e$–fatigue life relation is a geometry and load independent equation for “low stress-long life” loading with a small plastic zone at the crack tip. The $K_e$ approach is much simpler than the $J_e$ approach [2], since the former skips the detailed FE analyses. This simplicity would be quite beneficial to the analyses of multi-spot-welded structures [10,11].

2. Material properties of spot-welded specimens

2.1. Base metal properties

SPRC35 of this study is high-strength rolled steel used for the body and main shell-type components of automobiles. Its chemical contents are (C, Si, P, S, Fe) = (0.1, 0.05, 0.1, 0.04, balance: %). Two kinds of tensile specimens with a thickness of 1.0mm (1.0$t$) and 1.4mm (1.4$t$) are taken in the longitudinal and lateral
directions following the ASTM standard [12]. Fig. 2 shows the true stress–true strain curves of 1.4\textit{t} for two directions from MTS tensile tester with a quasi-static speed (0.5 mm/s). As the two curves differ 1.5% in maximum tensile strength and 3% in elongation, we made all the spot-welded specimens so that tensile loading may be longitudinally applied. If the true stress–true strain relation of base metal SPRC35 is decomposed into elastic (\(\sigma = E\varepsilon_e\)) and (\(\sigma = K\varepsilon_p^n\)) regions, the strain hardening exponent becomes \(n = 0.2\). The elastic modulus, strength modulus and yield and tensile strengths are (\(E, K, \sigma_y, \sigma_t\)) = (200 \times 10^3, 657, 245, 365) MPa.

### 2.2. Optimal welding condition

Spot-weld fails in either (pull-out) a button or an interfacial type. The latter occurs when the nugget forms incompletely due to insufficient electric current and pressure by abnormal surface condition. Hence sound spot-weld should fail in the button mode. JIS Z-3140 [13] presents the ideal nugget diameter for a given panel thickness. Here the ideal nugget diameter means the diameter, which guarantees the button type failure. JIS Z-3140 is a compilation of experimental investigations on factors determining failure types. The ideal nugget diameters of 1.0\textit{t} and 1.4\textit{t} are about 5 and 5.9 mm, respectively, that is, \(d = 5\sqrt{t}\). These ideal nugget diameters are valid for both twofold and threefold panels. We used the Air Spot Projection Welder. With dozens of overload failure tests, we found the optimal values of the electricity, pressure and current cycle, which yield the ideal nugget diameters. These optimal values are summarized in Table 1. The zinc braze zone [14] makes it difficult to demarcate the completely welded region. The maximum and minimum values of measured diameters thus differed about 5%. Their average value was taken as the nugget diameter.

### 2.3. Hardness distributions in the weld-zone

Fig. 3 shows the hardness distributions in the weld-zone. MicroVickers hardness tester was applied with a load 2.94 N to the points regularly distanced along the thickness and width directions of the specimen.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Welding conditions and diameters of 2 and 3 sheets spot-welds</th>
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<tr>
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<td>2 Sheets spot-welds</td>
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<tr>
<td></td>
<td>1.0\textit{t}</td>
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<tr>
<td>Welding current (kA)</td>
<td>8.9–9.1</td>
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<tr>
<td>Applied force (kgf)</td>
<td>300</td>
</tr>
<tr>
<td>Resistance welding time (cycles)</td>
<td>10</td>
</tr>
<tr>
<td>Nugget diameter (mm)</td>
<td>4.9–5.0</td>
</tr>
</tbody>
</table>
Here $y$ is the distance from the panel interface in the thickness direction, and $r$ is the distance from the nugget center in the longitudinal direction. The welding zone is sectioned into base metal (BM), heat-affected zone (HAZ), and fusion zone (FZ) based on the hardness value. Eqs. (1a) and (1b) [15,14] connect Vickers hardness $H_V$ to yield and tensile strengths. Here $n$ is the strain hardening exponent ($n = 0.2$ in SPRC35). Substituting averaging hardness values into Eqs. (1a) and (1b) produces yield and tensile strengths of the base metal, heat effected zone and fusion zone. Table 2 presents measured hardness, and calculated yield and tensile strengths. The calculated yield (tensile) strength is about 10% higher than that from the tensile test (Fig. 2). With calculated values as references, yield (tensile) strengths of HAZ and FZ are 38% and 76% higher than those of BM, respectively. This heat-affection induced hardening of the spot-welded part is the main cause of fatigue crack growth in the panel thickness direction, not in the nugget interfacial direction. We briefly discuss crack kinking in Section 4.3.

$$\sigma_y \text{ [MPa]} = 2.3 \times 10^{-n} H_V$$  \hspace{1cm} (1a)

$$\sigma_{ts} \text{ [MPa]} = 2.38(1 - n) \left( \frac{12.5n}{1 - n} \right)^n H_V$$  \hspace{1cm} (1b)

### 3. Limit load equations and overload failure curve

#### 3.1. Load–deflection curves of single spot-welded specimens

Four types of single spot-welded specimens were used in quasi-static overload failure tests (coach-peel: CP, cross-tension: CT, pure-shear: PS, tensile-shear: TS). CT–CP are opening modes, PS is a pure shear mode and TS is a mixed tensile-shear mode. Fig. 1 shows the shapes and dimensions of specimens following
KS [16]. The thicknesses of each specimen are $1.0t$ and $1.4t$. They automatically determine the nugget diameters (5, 5.9mm) guaranteeing the pull-out type failure as described in Section 2.2. The MTS cross-head speed was set as 0.5mm/s for quasi-static loading.

Figs. 4 and 5 show the load–deflection curves of CP–CT opening mode specimens. The CP specimen looks apparently half of the CT specimen, but the CP limit loads are about 70% (1.0$t$), 50% (1.4$t$) of halves of CT limit loads. The different limit loads, in spite of the same opening mode specimens, result from the discrepancy in degree of freedom. The CP specimen is free to move in the normal direction to the loading, while the CT specimen is fixed in that direction by four-end jigs. The failure pattern of the CP–CT specimen fails in the button type, where ductile failure occurs along the nugget periphery in the panel thickness direction by the local large deformation concentrated along the nugget periphery. Fig. 6 shows load–deflection curves of the 1.4$t$ CP specimen with various lengths of the moment arm $l_m$. This reveals that the CP limit load is independent of $l_m$. The unique deformation characteristic of the CP specimen explains this. Rotation of the moment arm part with increasing loads shortens the effective moment arm. At the final failure stage, the CP specimen straightens in a gull-wing form so that the effective moment arm nearly disappears. Consequently, at the time of failure, the normal stress on the nugget circumferential surface by the bending moment no more exists, and the shear stress on that surface by the longitudinal load remains as a dominant

![Fig. 4. Load–deflection curves of CP specimens.](image1)

![Fig. 5. Load–deflection curves of CT specimens.](image2)
factor. While nugget periphery fails eccentrically in the CP specimen, it fails evenly in the CT specimen due to geometry and load symmetries.

The PS specimen in a pure shear mode is presumed to fail in an interfacial manner. The spot point was, however, optimally formed to guarantee the button type failure. As a result of this, the PS specimen fails in neither the interfacial nor the button type. It is observed that a through thickness crack at the nugget center, which forms by initial thickness necking followed by ductile fracture, propagates in the specimen width direction so that the whole specimen thoroughly fails. Fig. 7 shows load–deflection curves of PS specimens. Here loads maintain certain values even after limit loads due to gradual cracking in the width direction, unlike CT–CP cases with sharp a load drop after limit load. Fig. 8 shows that the PS limit load is independent of specimen width.

Fig. 9 shows load–deflection curves of TS specimens. The TS specimen starts its failure with a semi-circular button mode preceded by thickness necking at the nugget center, then the neighboring base metal tears in an elliptical form having the major radius 3 times larger than the nugget radius. At the initial stage of loading, the nugget of the TS specimen rotates by couple forces, which are a panel thickness apart. At the moment of failure, the rotation angle reaches its maximum, and the couple forces come to lie on a single loading line. The maximum rotation angles are \( \theta \approx 20^\circ \), 15\( ^\circ \) for 1.0\( t \), 1.4\( t \), respectively, with a specimen width of 40mm. We may decompose the load on the nugget interface into tensile force \( P_n (= P \sin \theta) \) normal to its interface and shear force \( P_s (= P \cos \theta) \) parallel to the interface. Then \( P_n \) is the main factor for pull-out
failure by producing shear stresses around the nugget circumferential surface. Fig. 8 demonstrates that the TS limit load is linearly proportional to the specimen width, at least within the measured range of width. The rotational stiffness of the TS specimen increases with specimen width. The wider specimen thus restricts the rotation angle more, which reduces the portion of $P_n$ in $P$. If we assume that a failure occurs at the critical $P_n$, the wider specimen fails at the greater force.

3.2. Modal limit loads of a single spot-weld

The limit loads of a single spot-welded specimen can be expressed in terms of material properties of the heat-affected zone (HAZ), and geometrical parameters such as specimen thickness, width and nugget diameter. Only three types of spot-weld specimens (CT, PS, TS) were considered, since the CP specimen is less practical. For each load type (opening, pure-shear, mixed modes), we additionally tested the specimens of thickness $0.8t$ to express the limit loads in a quadratic for the thickness. The nugget diameter of the $0.8t$ specimen was naturally $4.5 \text{mm} (d = 5\sqrt{t})$ to guarantee a button type fracture.

We suggest the following three limit load equations (2a)–(2c) from our experimental results. As a result of heat-affected hardening, the HAZ of the dominant failure region exhibits higher strength distribution than the base metal yield strength. Considering this heat-affection-induced hardening, we attach a compensating bracket term to each equation. The use of an independent variable $t/d$ was motivated from the fact...
that the resistance heat energy applied to the spot region is proportional to the thickness \( t \), but resistance heat energy per unit nugget perimeter area is inversely proportional to the nugget diameter. It was stated above that only the limit loads of the TS specimen depend on the specimen width. A parameter \( \frac{w}{t} \) was taken here to describe the effect of width on the limit load, since the degree of width effect on the limit load decreases as \( t \) increases.

\[
P_{\text{lim}}_{\text{CT}} = P_n = \sigma_y \pi d t [-11.5 + 122(t/d) - 284(t/d)^2]
\]

\[
P_{\text{lim}}_{PS} = P_s = \sigma_y \pi d t [-16.9 + 188(t/d) - 455(t/d)^2]
\]

\[
P_{\text{lim}}_{TS} = P_{ts} = \sigma_y \pi d t [0.0106(w/t) + 1.304]
\]

3.3. Overload failure curve

Generally commercial finite element analysis programs describe the failure of spot point by the technique of nodal force release [5]. That is, the spot point is located at the nodes of the finite element model, and two nodes are constrained by a rigid element. When the reaction forces acting on the two nodes exceed the failure criteria, the constraint is released to simulate the failure of the spot. In that case, macroscopic failure criteria for spot-welds are given in the form of Eq. (3) [3]. Here \( P_n \) and \( P_s \) are tensile and shear forces, and \( P_{\text{lim}} \) are maximum (limit) tensile and shear force. In Eq. (3), only three variables \( (P_n, P_s, \alpha) \) are to be determined from the experiments.

\[
\left( \frac{P_n}{P_{\text{lim}}} \right)^2 + \alpha \left( \frac{P_n}{P_{\text{lim}}} \right) \left( \frac{P_s}{P_{\text{lim}}} \right) + \left( \frac{P_s}{P_{\text{lim}}} \right)^2 = 1
\]

Fig. 10 shows the failure curves generated from our experimental data points for two specimen thicknesses and three specimen widths. The details of our overload experiments were described in Section 3.1. Fig. 11 shows the failure curves normalized in the form of Eq. (3). All three curves can be approximated with a master curve without regard to specimen configuration. The value of \( \alpha \) in the master curve is 1.603, and the error between the master curve and the other three data curve is less than 3%. The limit load equations for \( P_n \) and \( P_s \) were presented in Section 3.2.

Fig. 10. Failure curves generated from the experimental data points \( t = 1 \) and 1.4mm, and \( w = 30, 40, 50 \)mm.
4. Load–fatigue life relation

We obtained raw load–fatigue life data of single spot-welded specimens from the fatigue tests for various loads with magnitudes sufficiently smaller than the specimen limit loads. With these data, the effects on fatigue lives of the loading mode and amplitude, and geometry of single spot-welded specimens are investigated in this section. We performed fatigue tests using the MTS test machine with the zero load ratio \( \frac{P_{\text{min}}}{P_{\text{max}}} = \text{minimum load/maximum load} \). Jigs and grips are the same as in the overload failure tests.

It is unrealistically difficult to exactly measure the crack initiation and growth occurring on the nugget front in a non-destructive way during the fatigue test. Many ways were suggested to estimate the crack initiation and growth with external data [16]. One is to measure the crack initiation with strain variation sensed by a special strain gauge attached on the nugget. In this case, the sensed strain variation is very small, and a variety of dynamic loads act as perturbation to the gauge so that the measured strain variation may have a significant noise error. Another is to estimate the crack growth through the measurement of crack opening displacement by a COD gauge. However, holes are needed to hold the gauge, and the resultant stress concentration on the hole affects the nugget front stress field. It is also hard to attach the gauge to the rotating TS specimen. Further, deformation exceeding the gauge range is not allowable.

The fatigue test is performed either by displacement or load control. In the former, where a fixed displacement is repeatedly applied, the load drops with the crack growth. In the latter, where a fixed load is repeatedly applied, the corresponding displacement is constant until crack initiation. At the moment of specimen failure, the displacement rapidly increases with crack growth. Fig. 12 shows the change of specimen displacement with the cycle number in a load controlled (500 N) fatigue test of the TS specimen. Here, the slopes of portions AB and BC are so obviously different that point B can be regarded as a crack initiation point. In portion BC, the crack grows in the thickness direction, and then the specimen completely fails at point C. Portion AB to the crack initiation is much greater than portion BC. In other words, portion AB dominates the cycles up to the whole failure. Therefore, we take the cycle number to point B, which is an estimation of the crack initiation point, as the fatigue life \( N_f \) of the spot-welded specimen in this study.

4.1. Fatigue behavior of single spot-welded specimens

Fig. 13 presents load–fatigue life data from the fatigue tests of CP-CT specimens. A fatigue crack starting at the nugget front center propagates in the thickness direction and on penetrating the thickness,
crack grows to a certain point along the nugget circumference and then advances in the specimen width direction. From the viewpoint of fracture mechanics, for a homogeneous specimen under mode I loading, the crack grows in the direction normal to the loading, but the cracks in the CP–CT specimens grow in the panel thickness direction rather than follow the nugget interface. This comes from the inhomogeneity of the welded part by heat-affection induced material hardening.

Fig. 14 presents load–fatigue life data from the fatigue tests of PS specimens. The crack in the PS specimen also grows in the specimen thickness direction as in CP–CT specimens. On penetrating the thickness, the crack grows along the nugget circumference and then advances in the specimen width direction, and leads the specimen to final failure. It is interesting to compare the fractured specimens from overload and fatigue tests. An overloaded specimen thins by necking from notable plastic deformation, and eventuates in sharp blade edges. Fatigued specimens failed under “high load-short life” conditions exhibited the features similar to those of overloaded specimens. On the other hand, fatigued specimens under “low load-long life” conditions underwent more brittle type deformation with limited plasticity. The cracks grew in the thickness and width directions without thinning. A fatigue-failed specimen under low load exhibited a granulated surface, as the crack spread against grains.

Fig. 15 presents load–fatigue life data from our fatigue tests of the TS specimens of mixed mode. The crack growth pattern in the TS specimens of low load-long life cases is similar to those of PS specimen.
On the other hand, the crack in TS specimens under high load starts a bit apart from the nugget, and advances in the specimen width direction. Local necking is observed around the nugget as in overload specimens. Fig. 16 displays the crack growth patterns observed on the fractured surfaces of four spot-welded specimens in “high load-short life” and “low load-long life” cases of our study.

4.2. Equivalent stress intensity factor $K_e$

In a fatigue test of spot-welded specimen, specimen deformation is very small because the applied load is much lower than the limit load, while local deformation occurs around the nugget in an overload test. Particularly, under “low stress-long life” loading, the plastic zone size near the nugget is relatively small, linear elastic stress intensity factor $K$ can thus be used for the fatigue life evaluation of spot-welded specimens. Fracture parameter $K$ carries dual implications such as an energy release rate and an intensity of crack-tip stress field.

As an energy release rate, the fracture parameter assumes that an advancing crack in homogeneous (at least in the crack direction) material is self-similar. That is, a crack advances in the same direction with the initial crack. The external crack of a spot-weld, which is the target of this study, however, advances in the panel thickness direction rather than following the nugget interface under both overload and cyclic load.
Therefore, fracture parameter $K$ as an energy release rate is inappropriate in this study. Another meaning of $K$ is the intensity of near crack tip (HRR) stress field, $J = K^2 / E'$ measuring the material damage at the crack tip. Here $E'$ is the effective plane strain Young's modulus. In other words, $K$ at the edge of the nugget is interpreted as a measure of the local material damage irrespective of the crack growth direction and fracture location in a spot-welded specimen. In this study, the fracture parameter $K$ is taken as its second implication.

Fig. 17 schematically illustrates the loads and moment applied to the nugget interface of a spot-welded specimen. The load $P$ is decomposed into tensile force $P_n$ and shear force $P_s$. Further, moment $M = P_n l_m$ due to the eccentric load is also applied on the center of the nugget section. Here $l_m$ is the distance from the center of the nugget section to the loading point. If the space between spot welds in a panel structure is wide enough, each spot-weld can be approximated as a circular joint zone connecting two infinite plates. In this case, stress intensity factors, $K_t$, $K_m$, $K_s$ due to tensile force, moment and shear force can be determined from the following relations [17]:

$$K_t = P_n / \left( d \sqrt{\pi d / 2} \right)$$  \hspace{1cm} (4a)

$$K_m = 6M / \left( d^2 \sqrt{\pi d / 2} \right)$$  \hspace{1cm} (4b)

$$K_s = P_s / \left( d \sqrt{\pi d / 2} \right)$$  \hspace{1cm} (4c)

Fig. 17. Axial ($P_n$) shear ($P_s$) forces and bending moment ($M$) sustained by a spot-weld.
In CP–CT tensile mode specimens, tensile force and moment act on the nugget interface. In the PS shear mode specimen, only shear force acts on the nugget interface. In the TS mixed mode specimen, tensile force, moment and shear force act all together. As the rotation angle of the TS specimen differs from load to load, we take half of the maximum rotation angle obtained from the overload failure test as an average rotation angle. Tensile and shear forces on the nugget interface are determined based on this average rotation angle. Note that the initial moment arm in the TS specimen is much smaller than that of the CP or the CT specimen, and it also decreases with the nugget rotation by load increase. Moment in the TS specimen is consequently negligible compared to tensile and shear forces. By the superposition principle, \( K_I \) and \( K_{II} \) are expressed as

\[
K_I = K_1 + K_m; \quad K_{II} = K_s
\]  

(5)

In the study of the fracture criterion and crack growth direction under mixed mode loading, fracture is assumed to occur when a fracture function \( f(K_I, K_{II}, K_{III}) \) reaches its critical value \( f_c \). The functional forms of \( f \) and \( f_c \) are contrived in many ways. First of all, from the viewpoint of the \( f_c \) energy balance criterion, fracture occurs when total energy release rate, \( G = G_I + G_{II} \), reaches the critical value. As \( G_I = K_{IC}^2 / E \), \( G_{II} = K_{II}^2 / E' \) in mode I and II loading, respectively, the following mixed mode fracture criterion can be suggested.

\[
K_I^2 + K_{II}^2 = \text{constant} = K_{IC}^2
\]  

(6)

Since \( K_{II} = 0 \) for mode I cracking, \( K_I^2 = K_{IC}^2 \). Since \( K_I = 0 \) for mode II cracking, \( K_{II}^2 = K_{IC}^2 \). Consequently Eq. (6) requires \( K_{IC} = K_{IC} \), which means that the locus for combined mode cracking is a circle of radius \( K_{IC} \). In experimental practice, however, \( K_{IC} \neq K_{IC} \). Hence, the fracture criterion would more likely to be \( [18] \)

\[
\left( \frac{K_I}{K_{IC}} \right)^2 + \left( \frac{K_{II}}{K_{IC}} \right)^2 = 1 \iff K_I^2 + \left( \frac{K_{IC}}{K_{IC}} \right)^2 K_{II}^2 = K_{IC}^2
\]

(7)

The energy balance criterion (7) is based on the assumption that a crack propagates in a self-similar manner. In other words, it is assumed that crack extension is in the plane of the original crack. In mixed mode experiments, it is usually observed that a crack tends to propagate in the direction slanted from the original cracked plane. This invalidates the assumption. The energy release rate criterion can be modified in such a way that a crack will grow in the direction of maximum energy release rate. Such a criterion can be shown to be equivalent to the maximum tangential stress (MTS) fracture criterion proposed by Erdogan and Sih [19]. The MTS fracture criterion postulates that a crack will grow in the normal direction to the maximum tangential stress. At the crack tip under mixed mode loading, let \( \theta_m \) be an angular position of the maximum tangential stress measured from the original crack plane. The angle \( \theta_m \) is determined from \( \tau_{r0} = 0 \) or equivalently \( \partial \sigma_{r\theta} / \partial \theta = 0 \) condition. When \( \sigma_{r\theta}(\theta_m) \) is related to fracture stress, \( \sigma_f(\theta_m) = \sqrt{K_{IC} / \sqrt{2 \pi r}} \) of pure mode I loading, the following fracture criterion for mixed mode loading is established [19].

\[
K_I \cos^3 \frac{\theta_m}{2} - 3K_{II} \cos^2 \frac{\theta_m}{2} \sin \frac{\theta_m}{2} = K_{IC}
\]  

(8)

With reference to the functional forms of Eqs. (7) and (8), the \( K_{IC} \) based fracture criterion for mixed mode loading could be generalized as

\[
K_{II}^2 = f_1(K_I^2) + f_2(K_1, K_{II}) + f_3(K_{III}) = K_{IC}^2
\]

(9)

Letting \( f_2 = 0 \) in Eq. (9) and revising Eq. (7) Broek [20] suggested a simplified equivalent stress intensity factor \( K_c \), and corresponding fracture criterion as in Eq. (10). Here \( \beta \) is a material constant, which measures the material sensitivity on mode II loading and is closely related to the ductility of the material. The next section discusses this matter in detail.
\[ K_e^2 = K_1^2 + \beta K_{II}^2 (= K_{IC}^2); \quad \beta = \left( \frac{K_{IC}}{K_{II}} \right)^2 \] (10)

4.3. Fracture direction under mixed mode loading

It is very difficult to predict the crack growth direction under mixed mode loading because, as mentioned above, a crack does not grow in a self-similar manner, and even once initiated cracks could keep changing its propagation direction. On this matter, Melin [21] studied the crack growth direction with a kinked crack model under a mixed mode load, and showed that the ratio \( \frac{K_{II}}{K_{IC}} \) is the main factor determining the crack growth direction. Fig. 18 illustrates a kink of infinitesimal length forming an angle \( \alpha \) with the main crack. Under mixed mode loading, stress intensity factors at the tip of infinitesimal kink \( k_I \) (in mode I) and \( k_{II} \) (in mode II) can be calculated in terms of the stress intensity factors \( K_I \) and \( K_{II} \) at the tip of the main crack prior to kinking [22] as in Eq. (11). Here is the tilting angle of the kinked crack, and \( R_{ij} \) consist of trigonometric functions of \( \alpha \).

\[
k_I(\alpha) = R_{11}(\alpha)K_I + R_{12}(\alpha)K_{II}
\]
\[
k_{II}(\alpha) = R_{12}(\alpha)K_I + R_{22}(\alpha)K_{II}
\]

Incipient kinking is assumed to occur in the direction that maximizes stress intensities at the tip of the infinitesimal kink [21]. Namely, the crack advances in the direction making \( k_I(\alpha) = \alpha_{max} \) for mode I fracture,
and advances in the direction making $k_{II}(a) = k_{I_{\text{max}}}$ for mode II fracture. Two curves in Fig. 19, theoretically obtained from Eq. (11), indicate the values of $K_I$ and $K_{I_{\text{II}}}$ necessary to initiate incipient kinking for mode I and mode II fracture, respectively. Under mode I fracture criterion (assumption), even if pure mode II load is applied, mode I fracture should occur. Therefore $K_{I_{\text{II}}} = 0.81K_{IC} < K_{I_{\text{IC}}}$. That is, for mode I fracture to occur under pure mode II loading, $K_{I_{\text{IC}}}/K_{IC}$ should be greater than 0.81. On the contrary, for mode

![Fig. 20. Equivalent stress intensity factor vs. failure cycles in modes I and II and mixed mode for (a) $\beta = 0.1$, (b) $\beta = 1.0$, (c) $\beta = 4.3$, (d) $\beta = 30$ (logscale).](image-url)
II fracture to occur, $K_{IIc}/K_{IC}$ should be less than 0.81. In the same context, for incipient mode II fracture under mode II loading, $K_{IIc}/K_{IC}$ should be less than 0.38. In conclusion, in order for mode II fracture to occur, $K_{IIc}/K_{IC}$ should be less than the value between 0.38 and 0.81. But in common materials, as $K_{IIc}/K_{IC}$ is greater than 1, mode II fracture seldom occurs even under pure mode II loading. From the relation between the material constant $\beta$ and $K_{IIc}/K_{IC}$, it can be inferred that for $\beta$ greater than the value between 1.5 and 7, mode II fracture could occur [21].

Hallback [23] demonstrated that the fracture angle under mixed mode loading varies with ductility of a material. Hallback suggested maximum shear stress (MSS) fracture criterion implying that a crack advances along the direction of maximum shear stress, and verified the effectiveness of MSS fracture criterion via mixed mode fracture tests of aluminum alloy 7075-T6. MTS fracture criterion successfully explains the fracture direction of a brittle material like PMMA under mixed load loading, but incurs a large difference between predicted and actual values in the case of ductile materials. Under pure mode II loading, a crack advances with a fracture angle of $-70^\circ$ in PMMA, but in aluminum alloy, advances with almost $0^\circ$, i.e., in the direction predicted by the MSS fracture criterion. This indicates that ductility of a material is an important factor, which determines the fracture mode. As $K_{IIc}/K_{IC}$ is a material constant ($=\beta^{-0.5}$) commanding fracture mode, it is related to “material sensitivity on mode II loading”, that is, “ductility of material”. Pook [24] found a relation, showing that $K_{IIc} \approx 0.75K_{IC}$, through mixed mode fracture tests with aluminum alloy DTD 5050. In such a case, material constant $\beta$ for aluminum DTD 5050 is equal to 1.78.

### 4.4. Relation between $K_e$ and fatigue life

In this section, equivalent stress intensity factor defined by Eq. (10) is adopted as a (crack-driving) fatigue life prediction parameter of spot-welded specimens. The material constant $\beta$ is calibrated in the process of recasting the raw load–fatigue life tests data of Figs. 13–15 into the $K_e$–fatigue life relation. Thereby it is possible to more sharply define the $K_e$–fatigue life relationship. To this, selecting coefficients $A_i$ ($i = 1, 2$) and $\beta$ in the linear regression of $\log K_e = A_1 + A_2 \log N_f$ as three design parameters, and difference between regression and data as an objective function, we adopted an optimization technique. Here, as $K_e$ itself is a function of $\beta$ as in Eq. (12), finding optimum coefficients $A_i$ and $\beta$ turns out to be essentially a non-linear optimization problem. By repetitive calculations of design parameters until variation of objective function becomes less than the tolerance 0.01, an optimum value $\beta = 4.3$ was obtained. As stated above, this value can be regarded as the material constant measuring ductility of SPRC 35 cold-rolled steel panels used in this study. Fig. 20 shows the degree of data dispersion from the $K_e$–$N_f$ regression for the value $\beta$ varying from 0.1 to 30. As the value of $\beta$ increases from 0.1 to 0.3, $K_e - N_f$ test data is densely placed around the regression curve, and as the value exceeds the optimal value 4.3, the distribution becomes wider. From $K_e - N_f$ data of Fig. 20(c), the solid line is given as (12), where $(A_1, A_2) = (2.28, -0.20)$.

$$\Delta K_e [\text{MPa} \cdot \text{m}] = 10^{41} N_f^{41}$$

(12)

The $K_e$-fatigue life relation is a geometry and load independent equation for “low stress-long life” loading with a small plastic zone at the crack tip. The $K_e$ approach is much simpler than the $J_e$ approach [2], since the former skips the detailed FE analyses. This simplicity would be quite beneficial to the analyses of multi-spot-welded structures [10,11].

### 5. Concluding remarks

In this study the quasi-static load–displacement curves and raw load–fatigue life data are obtained via the quasi-static overload and fatigue tests for four types of spot-welded specimens. The overload failure
curve is presented. The limit loads of spot-welded specimens are expressed in terms of yield strength of base metal and geometric dimensions. Recasting the load–fatigue life raw data in terms of equivalent stress intensity factor, a comprehensive equation for fatigue life predication is suggested. The following concluding remarks can be made from this study.

1) The over load failure criterion of a single spot-weld specimen can be given as a master curve in a mixed mode domain, the coordinates of which are normalized by limit loads. The limit load of a single spot-welded specimen can be expressed as (3a)–(3c) in terms of material properties of the heat-affected zone (HAZ), and geometrical parameters such as specimen thickness, width and nugget diameter.

2) CT–CP opening mode specimens fail primarily with the shear stresses on the nugget circumferential surface. The PS shear mode specimen fails primarily with the tensile stresses on the nugget circumferential surface. While the PS limit load is independent of the specimen width, the TS limit load linearly increases with specimen width. The increase in rotational stiffness of the TS specimen with specimen width explains this.

3) For spot-welded specimens under low stress-long life condition, we suggested an inclusive fatigue life equation in the form of $D_{K}e = 10^{A_1}N_2^{b}$ where $K_2^e = K_2^I + \beta K_2^{II}$ and $\beta$ is a constant measuring the “material sensitivity” on mode II loading, or the “ductility” of the material.

Acknowledgement

The authors are grateful for the support provided by a grant from the Korea Science and Engineering Foundation (Grant No. KOSEF 97-0200-0501-3).

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