Overload analysis and fatigue life prediction of spot-welded specimens using an effective $J$-integral

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Abstract

This paper proposes an integrated approach for predicting the fatigue life of spot-weld specimens. The approach is independent of specimen geometry and loading type. We establish finite element (FE) models reflecting the actual specimen behavior observed from the experimental load-deflection curves of four types of single spot-welded specimen. Using established FE models, we first evaluate $J$-integral as a fracture parameter for describing the effects of specimen geometry and loading type on the fatigue life. It is confirmed, however, that $J$-integral concept alone is insufficient to produce the generalized relationship between load and fatigue life of spot-welded specimens. On this ground, we introduce another effective parameter $J_e$ composed of $J_I$, $J_{II}$, $J_{III}$. The mechanism of the mixed mode fracture, which incites the use of $J_e$, is discussed. The parameter $J_e$ is demonstrated to more sharply define the relation between load and fatigue life of spot-welded specimens. The crack surface displacement method is adopted for decomposition of $J_e$.

Keywords: Spot-welded specimen; Overload analysis; Fatigue analysis; Finite element model; $J$-integral; Effective crack driving parameter $J_e$; Fatigue life

1. Introduction

Spot-welding is a method of joining shell-type steel structures. It is effective in view of auto-lightening, since it only needs resistance heat without any additional materials. Durability, consistent weld-quality and high productivity from automation further allow the spot-welding to be used for joining structural panels of railway vehicles and home appliances. Understanding of overload and fatigue behaviors of spot-welds is important in structural and crashworthy analyses, in addition to the knowledge on the mechanical behavior of plate itself. The overload and fatigue behaviors of spot-welds are perceived as one of the toughest problems in solid mechanics. First, large deformation with substantial geometry change precedes the fracture of spot-welded ductile materials such as high strength steel panel for auto-structures. Hence numerous 3-D continuum finite elements (FEs) need to be used around the spot-weld for accurate deformation analyses. Second, external crack-type configuration of spot-weld requires an approach with fracture mechanics...
in a strict sense (Pook, 1975a,b; Wang and Ewing, 1988, 1991; Lee and Choi, 2004; Lee and Kim, 2004; Lee et al., 2003). On top of that, strength variation due to resistant heat around spot nugget should be additionally considered (Lee and Choi, 2004; Lee and Kim, 2004; Lee et al., 2003; Shepard, 1993; Zuniga, 1994). Recently, enhancements in computation speed and FE software have brought some progress on spot-weld research (Lee et al., 1998; Shin et al., 1999).

Failure analyses of spot-welds are largely divided into two; the (quasi-static) overload failure analysis and the fatigue analysis. The primary aim of overload analysis is to provide a failure curve, which facilitates the macroscopic FE analyses of car crashworthiness (Lee et al., 1998; Shin et al., 1999). The failure curve is usually presented in mixed mode force domain in terms of material property and some geometrical quantities such as nugget diameter and panel thickness. Fatigue analysis of spot-welded specimens may demand a lot of time and effort in collecting the fatigue data. Once the test data are obtained, however, the fatigue failure can be predicted by a fracture parameter describing the effects of loading type and specimen geometry in an inclusive manner.

While spot-welding is generally used in the form of multi-spots, the fatigue strength of multi-spot-welded structure is eventually determined by the fatigue strength of each single spot-weld. Therefore, examining fatigue strength of various types of single spot-welded specimens is essential for the assessment and design of spot-welded panel structures.

Pook (1975a,b) first attempted the fracture mechanics approach for fatigue analysis of spot-welded specimens with an external crack shape of spot-weld in mind. He showed that experimental data from tensile-shear specimens become much less scattered, and fatigue limit is more clearly defined by recasting the load–fatigue life relation to stress intensity factor-fatigue life relation. However, only opening-mode stress intensity factor $K_I$ was taken as a crack driving parameter in spite of mixed-mode loading in tensile-shear specimens. The validity of Pook’s method is thus limited to very low magnitude of applied load and small scale-yielding (SSY) condition. To overcome this limitation, Wang and Ewing (1988, 1991) selected an elastic–plastic fracture parameter, $J$-integral, as fatigue crack growth-driving parameter. They inclusively described the effects of material, specimen configuration using a single parameter $J$. But their work yet lacks practical use, since their fatigue life prediction depends on the loading mode; tensile-shear (Wang and Ewing, 1988) or pure shear (Wang and Ewing, 1991).

On the above background, this study suggests an inclusive fatigue life equation, which is independent of specimen geometry and loading type, for single-spot-welded specimens. FE models are established after the load–deflection curves from overload tests of single-spot-welded specimens (Lee et al., 2003). We first obtain the $J$-distribution along the nugget front via elastic–plastic FE analyses. The location of maximum $J$-value ($J_{max}$) can be regarded as a failure initiation point. The modal values $J_I$, $J_{II}$, $J_{III}$ of $J_{max}$ are obtained using the ratios (Matos et al., 1989) of stress intensity factors from elastic FE analyses under the same load magnitude with that of elastic–plastic FE analyses. These modal values produce an effective $J$-integral, denoted $J_e$. The mechanism of the mixed mode fracture, which motivated us to use of $J_e$, is also discussed. Finally the load–fatigue life data specimens (Lee et al., 2003) are recast in terms of this $J_e$. Comparison of predicted fatigue lives to experimental ones validates the suggested approach with $J_e$. The $J_e$ concept is similar to the equivalent stress intensity factor (Lee et al., 2003). As based on the elastic–plastic analyses, however, this $J_e$ approach more sharply predicts the fatigue life, especially in higher load cases.

2. Finite element model of spot-welded specimens

2.1. Distributions of material properties

Resistant heat produces inhomogeneous microstructures around the spot-nugget (Shepard, 1993; Zuniga, 1994). This inhomogeneous property should be reflected on FE model to embody the actual mechanical behavior of spot-weld. We thus establish three FE models of the heat-affected zone
(HAZ) based on the hardness distributions that were measured by Lee et al. (2003) with micro-Vickers hardness tester.

A model-1 by Zuniga (1994) consists of fusion zone (FZ), four heat-affected zones (HAZ) and base metal (BM) region. However, the preliminary FE analyses showed that this model is not generally applicable, since the model takes no notice of panel thickness and number of panels, which are the ultimate parameters for determining welding conditions. We thus consider the more practical model-2 (Fig. 1a) and model-3 (Fig. 2a) according to the number of welded-sheets (2–3-fold). Unlike Zuniga’s model-1, model-2 and model-3 are sectioned in thickness direction. It is noteworthy that even for a given model-2 or model-3, the size of each sub-region and plastic properties vary according to the panel thickness, which determines the welding conditions. In 3-fold PS specimen particularly, exterior and interior panel undergo quite different heat affection, which leads to dissimilar sub-regions. Path independency of $J$ (Anderson, 1995), a fatigue crack driving parameter, is assured by putting the external crack front (= nugget end) within a region where material is homogeneous in crack direction. Plastic properties of sub-regions of 1.0t model-2 and 1.4t model-3 of SPRC35 specimens (Lee et al., 2003) were determined with hardness-strength relation proposed by Zuniga (1994). That is, in stress-strain relation of base metal, the stress for given strain is amplified to give the stress–strain relation of sub-region by the hardness ratio between sub-region and base. Figs. 1b and 2b show the regional stress-strain relations of model-2 and model-3 of SPRC35 1.0t and 1.4t specimens.

Fig. 1. (a) The shape of HAZ model-2 of 1.0t specimens and (b) flow stress–plastic strain curves of HAZ model-2 of SPRC35 1.0t specimens.

Fig. 2. (a) The shape of HAZ model-3 of 1.4t specimens and (b) flow stress–plastic strain curves of HAZ model-3 of SPRC35 1.4t specimens.
2.2. FE modeling of single spot-welded specimens

This study aims to describe the experimentally obtained load–fatigue relation in terms of $J$. The configurations of four types of spot-welded specimens in fatigue experiment (Lee et al., 2003) are therefore selected as FE models (Fig. 3).

Two L-type panels welded by single-spot form a coach-peel (CP) specimen as in Fig. 3a. Only a quarter of CP specimen is modeled due to load and geometry symmetries (Fig. 4a). Nodal displacement in $y$-direction is fixed at $xz$-symmetry plane of Fig. 4a. Loading displacement in the $z$-direction is applied to the loading surface of elbow end, and the $z$-directional displacement of nugget bottom surface is fixed. Contraction in the thickness direction is allowed by freeing the $x$-directional displacement at loading surface. Only $x$-directional displacement at the innermost edge is fixed to prevent the $x$-directional rigid body motion. Displacement in $y$-direction at loading surface, which is constrained by frictional force, is fixed. In the above boundary conditions, rotational degrees of freedom are not considered as 3-D continuum.

![Fig. 3. Geometries and loading modes of four types of spot-welded specimens (unit: mm).](image)

![Fig. 4. (a) Boundary conditions of CP specimens; (b) boundary conditions of CT specimens; (c) boundary conditions of TS specimens and (d) boundary conditions of PS specimens.](image)
finite elements are used. The FE model consists of about 1300 C3D20R elements (ABAQUS Library, 1998), and reduced integration is adopted to avoid the problem associated with incompressibility inside the plastic zone. To deal with the contact problem, 2200 rigid surface elements (ABAQUS Library, 1998) are also placed at the bottom surface of upper half of CP specimen model. All degrees of freedom of the rigid surface and the z-directional displacement of bottom surface are fixed. Meanwhile, the elastic–plastic porous material of Tvergaard (1981) is adopted in this study to describe the failure characteristics of CP specimen. We discuss this matter in Section 3.3.

Two crossed cruciform panels welded by a single-spot form a cruciform cross-tension (CT) specimen as in Fig. 3b. Only one eighth of CT specimen is modeled due to load and geometry symmetries (Fig. 4b). Nodal displacement in y-direction is fixed at xz-symmetry plane of Fig. 4b, and x-directional displacement is fixed at yz-symmetry plane. Loading displacement in z-direction is applied to the nugget bottom surface, while all the degrees of freedom are fixed at fixed end. Rotational degrees of freedom are not naturally considered as three-dimensional continuum finite elements are used. The FE model consists of about 700 C3D20R elements, and reduced integration is also adopted.

A half of tensile-shear (TS) specimen in Fig. 3c is modeled due to load and geometry symmetries (Fig. 4c). Nodal displacement in y-direction is fixed at xz-symmetry plane of Fig. 4c. The displacements in x- and y-directions are fixed at fixed end, while x-directional loading displacement is applied to loading end. Contraction in thickness direction is allowed by letting the z-directional displacement free at both ends. But the z-directional displacements at z = 0 edges of both ends are fixed to prevent the z-directional rigid body motion. The FE model consists of about 1300 C3D20 elements for each low and upper half of specimen, thus the total FE model consists of about 2600 elements. The nugget interface between two panels is joined by rigid beam elements (ABAQUS Library, 1998). Contrary to the general prediction, preliminary FE analyses showed that the complete integration provides more practical stiffness than the reduced integration in TS specimen. We thus adopt the complete integration elements in TS specimen models.

Single spot-welding of 3-folded panels together produces a pure-shear (PS) specimen as in Fig. 3d. A quarter of PS specimen is modeled due to load and geometry symmetries (Fig. 4d). The FE model and boundary conditions of PS specimen of Fig. 4d are basically similar to those of TS specimen (Fig. 4c) except fixed z-directional displacement at xy-symmetry plane of the inside panel. Inside panel is half-modeled in its thickness. Inside and outside panels come into contact as the applied load increases. To deal with this contact problem, 2000 rigid surface elements (ABAQUS Library, 1998) are also placed at the bottom surface of inside panel and top surface of outside panel. Complete integration provided again more practical stiffness than the reduced integration in PS specimen. We thus also adopt the complete integration elements in PS specimen models.

3. Quasi-static overload failure analyses

3.1. Failure behavior of CP specimen and porous model

Geometry of CP specimen induces its own deformation and failure characteristics. An undeformed CP specimen carries a moment arm between loading line and spot-nugget end. As applied eccentric load increases, the effective moment arm decreases by the rotation of initial moment arm part. The nugget front of CP specimen undergoes relatively concentrated tensile stress from the bending moment, while that of TS specimen undergoes uniform tensile stress from the tensile loading. Consequently, “ductile fracture” in thickness direction occurs close to the nugget in CP specimen, while local thickness necking occurs a bit apart from the nugget in TS (Zuniga, 1994). To describe this ductile fracture behavior of CP specimen, we overlap the elastic–plastic porous material of Tvergaard (1981) upon the sub-regions of model-2.

Ductile failure with void nucleation, growth and coalescence is in explicable with Mises or
Tresca yield criterion only. Gurson (1977) proposed a yield criterion for an elastic–plastic porous material

\[
\frac{\sigma_e^2}{\sigma_y^2} + 2f \cosh \frac{3\sigma_m}{2\sigma_y} - (1 + f^2) = 0
\]

(1)

Here \( f \) is the volume ratio of void part to entire material, \( \sigma_y \) yield strength of specimen, \( \sigma_e \) effective stress, and \( \sigma_m \) mean normal stress. Gurson model, however, fails to explain the localization occurring in the terminal stage of ductile failure. To supplement the model, Tvergaard (1981) proposed another yield criterion for an elastic–plastic porous material with variable material properties \( q_1, q_2 \)

\[
\frac{\sigma_e^2}{\sigma_y^2} + 2f \cosh \left( -q_2 \frac{3\sigma_m}{2\sigma_y} \right) - (1 + q_1^2f^2) = 0
\]

(2)

Here the value range of \( (q_1, q_2) \) generally taken are \( q_1 = 1.25–2.4 \) and \( q_2 = 0.75–1.0 \). In this work, we selected \( (q_1, q_2) = (1.7, 1) \) as the optimal values of SPRC35 rolled-steel by calibrating the FE solutions of load–deflection curve to experimental ones (Lee et al., 2003). The volume ratio increases as specimen deepens its deformation. The initial volume ratio \( f \) is taken as 0 and the \( f \) after void initiation is taken as 0.04 by considering the volume ratios of SPRC contents converted from the mass ratios. The parameters determining the void nucleation (ABAQUS, 1998), \( e_N \) and \( \varepsilon_N \) are selected as 0.125 and 0.055 respectively by adjusting the FE load–deflection curve of CP specimen to follow the experimental ones.

Fig. 5 compares the FE solutions of load–deflection curve of CP specimen to those from overload tests for two specimen thicknesses. Open circles indicate experimental curves (Lee et al., 2003); dotted and dashed lines are FE solutions with only model-1 and model-2 respectively; solid lines are FE solutions with model-2 and porous model superposed. It is shown that the FE loads increase after the spot-failures if only model-2 is used without the porous model. The FE loads drop after the limit loads if the porous model is superposed on the model-2 of HAZ; i.e., the post-peak FE solutions follow the test curves. For porous model, non-singular finite elements were used around nugget periphery for rapid numerical convergence. Note, however, that even model-2 alone provides a good load–deflection curve at least until the spot-failure. In chapter 4 on fatigue analyses, we thus use singular elements around model-2 nugget periphery for an efficient calculation of \( J \)-integral without porous model.

3.2. Load–displacement curves of CT specimen

Fig. 6 compares the FE solutions of load–deflection curve of CT specimen to those from overload tests. Open circles indicate experimental curves (Lee et al., 2003); solid lines indicate FE solutions with only model-2. CT specimen needs only model-2 without porous model, since it fails not by ductile fracture but by local thickness necking near nugget end. The plastic hinge occurring at the fixed end (Fig. 4b) of CT specimen makes its FE behavior sensitive to the mesh refinement. Sufficient refining of the plastic hinge region with large local deformation kept the FE specimen from over-stiffening. FE limit load solutions for 1t and 1.4t CT specimens almost coincide with the test values as shown in Fig. 6. Displacement values for a given load, however,
show the discrepancy widening with load increase. The discrepancy comes from the relative sliding between specimen and grip within the gap between joining bolt and hole. In this study, we neglect the relative sliding error, and refer only to the limit load value for FE model establishment. As the displacement discrepancies in TS and PS specimens below are also explained with the same reason, only limit load values are referred for FE model establishment.

3.3. Load–displacement curves of TS and PS specimens

Fig. 7 compares the FE solutions of load–deflection curve of TS specimen to those from overload tests for two specimen thicknesses. Open circles indicate experimental curves (Lee et al., 2003); solid lines indicate FE solutions with only model-2. The FE solutions agree well with the test curves. The HAZ model of TS specimen uses only model-2 without porous model as CT specimen case. Slight displacement discrepancy may come again from the relative sliding between specimen and grip. Fig. 8 is the comparison of FE load–deflection curve of PS specimen to those from overloads tests. Open circles are experimental curves (Lee et al., 2003); solid lines FE solutions with only model-2. The HAZ model of PS specimen also uses only model-2 without the porous model. Slight displacement discrepancy may also come again from the relative sliding between
specimen and grip. It is noteworthy that the PS load maintains its value even after peak load, unlike the sharp load-drop after limit load in CT–CP cases. It is due to gradual cracking in the width direction (Lee et al., 2003).

4. Fatigue failure analyses

The primary aim of fatigue analysis of spot-welded specimen is to predict the failure life in terms of applied load. On the contrary, in design point of view, the aim is to provide the basic data which determine maximum allowable load for a given failure life. The load–fatigue life relation of single spot-welded specimen carries an important meaning in that it is a basic unit for describing the fatigue strength of multi-spot-welded panel structures. The fatigue life of spot-welded specimen is governed not only by load magnitude but also by loading type and geometric variables. The geometric variables include nugget diameter, specimen thickness and specimen width. Considering only load magnitude (kN) without these additional geometric variables, we end up with totally pointless load–fatigue life relations as shown in Fig. 9. Consequently a crack driving parameter is required, which quantifies the effects of load magnitude, type and specimen geometry in an inclusive manner. To this, we first attempt to evaluate the $J$-integral as a fracture parameter for describing the effects of specimen geometry and loading type on the fatigue life, following Wang and Ewing (1988, 1991).

4.1. Path-independency of damage measurer $J$

In this study, the elastic–plastic fracture parameter $J$ is taken as a measure of the deformation measurer at the spot-nugget front, which is equivalent to an external crack. Fracture parameter $J$ carries dual meanings as an energy release rate and an intensity of crack-tip stress field (Anderson, 1995). As an energy release rate, fracture parameter $J$ assumes that an advancing crack in a homogeneous (at least in the crack direction) material is self-similar. That is, a crack advances in the same direction with the initial crack. The external crack of spot-weld, which is the target of this study, however, advances in the panel thickness direction rather than following the nugget interface under both overload and cyclic load. Therefore, fracture parameter $J$ as an energy release rate is inappropriate for spot-weld case. Another meaning of $J$ is the intensity of near crack-tip (HRR) stress field, measuring the material damage at the crack-tip. In other words, $J$ at the edge of nugget is interpreted as a measure of local material damage irrespective of crack growth direction and fracture location in a spot-welded specimen. In this study, the fracture parameter $J$ is taken as its second implication.

Resistant heat generates inhomogeneous microstructures around the spot-nugget (Shepard, 1993; Zuniga, 1994). To reflect the inhomogeneity on FE model, we established two FE models of the heat-affected zone (HAZ) in Section 2.1. Moreover, in embodying the actual mechanical behavior in FE analyses of spot-weld, the path-independency of $J$ (Anderson, 1995) should be assured in addition to inhomogeneous HAZ modeling. We put the external crack front (= nugget end) within a region where material is homogeneous in crack direction (Figs. 1a and 2a). Consequently, HAZ model-2 and model-3 proposed in Section 2.1

![Fig. 9. Correlation between load ($P$: kN) and fatigue life ($N_f$) for four types of spot-welded joints (Lee et al., 2003).](image-url)
guarantee the path-independency of $J$, even in higher loading. In this chapter 4 on fatigue analyses, for an efficient calculation of $J$-integral, we use singular elements around model-2 nugget periphery without superposing porous model. Note again that even model-2-alone provides a good load–deflection curve at least until the spot-failure (Fig. 5).

4.2. Fatigue failure analyses with $J$ alone

The $J$-values along nugget front are obtained by virtual crack extension/domain integral methods (ABAQUS, 1998; Parks, 1977; Shih et al., 1986). The $J$-distributions along nugget front vary considerably with loading type and specimen geometry, which in turn vary according to the type of spot-weld specimen (CP, CT, TS and PS). In CP specimen of pure tensile mode (Fig. 10a), maximum $J$ occurs at angular position $0^\circ$ which is the nearest to the loading plane, while $J = 0$ at angular position $180^\circ$ by crack closure. In CT–TS specimen (Fig. 10b–c), maximum $J$ also occurs at angular position $0^\circ$ and the $J$-distribution is symmetric with respect to angular position $90^\circ$. In 3-folded PS specimen (Fig. 10d), in contrast to other CP–CT–TS specimens, maximum $J$ occurs at angular position $90^\circ$ and minimum $J$ occurs at angular position $0^\circ$. The position of maximum $J$ (angular position $0^\circ$ or $90^\circ$) in Fig. 10 can be taken as the failure initiation point under fatigue loading. Wang and Ewing (1988, 1991) prescribed this maximum $J$ ($J_{\text{max}}$) as a load-induced damage measure of material, and then proposed a $J_{\text{max}}$–fatigue life relation for each loading type. Fig. 11 presents the $J_{\text{max}}$-fatigue life relations by recasting the load–fatigue life relations (Lee et al., 2003). We observe that $J_{\text{max}}$-fatigue life data are too scattered to be a general relation irrespective of loading type and specimen geometry.

4.3. Modal decomposition of $J$

Knowles and Sternburg (1972) generalized the scalar $J$ into the vector $\mathbf{J}$ to describe the crack extension in an arbitrary direction. In that case, the directional components $J_k$ of vector $\mathbf{J}$ are expressed as

![Fig. 10. Variations of normalized $J$ ($= Et^3J/P^2$) with angular position of spot-welded specimens.](image)
where $J_k = J_{\text{max}}$ is the maximum $J$ value under mixed-mode loading for the stress field $\sigma_{ij}$, and $J_{\text{max}}$ is the maximum $J$ value under mixed-mode loading for the stress field $\sigma_{ij}$.

Here axis-I is normal to crack line and axis-II is normal to crack plane (I–III plane), III axis is normal to crack plane ($\beta$), and Blackburn (1975) showed that a crack extends along axis-I, axis-II, and axis-III.

On this observation, we propose a novel crack driving parameter $J_{\text{d}}$ to describe the effects of load magnitude, type and specimen geometry on fatigue life of single spot-welded specimens in an inclusive manner. $J_{\text{d}}$ is composed of its modal components as in Eq. (1), and can be viewed as a measure of damage extent at the nugget front, irrespective of crack propagation direction.

To this, we first regard the maximum $J (J_{\text{max}})$ of Fig. 10 as the absolute value of $J$ as Eq. (4). Note again that the $J$, the intensity of HRR field deep inside the plastic zone, is a damage-measure at the nugget front. The following can be inferred from the geometry and loading type. The vector $J$ corresponding to $J_{\text{max}}$ consists of only $J_1$ in mode I CP–CT specimens. Further, the vector $J$ in mode I and II TS specimen consists of $J_1$ and $J_2$, and the vector $J$ in mode I and II PS specimen consists of $J_1$ and $J_2$.

$$|J| = \sqrt{J_1^2 + J_2^2 + J_3^2} = J_{\text{max}}$$

Using crack surface displacement method, Matos et al. (1989) obtained the modal $K_1$, and $K_II$ values under mixed mode loading for bimaterial. We thus perform elastic FE analyses on spot-welded specimen modes under the same load magnitude with that of elastic–plastic FE analyses. Then the modal values $K_1$, $K_II$, $K_III$ on the crack front are obtained with the following:

$$\Delta u_i = \frac{4(1-\nu)K_i}{G} \sqrt{\frac{r}{2\pi}}, \quad i = I, II, III$$

Here $G$ is shear modulus, $\nu$ Poisson’s ratio, $r$ the distance from the crack front, $\Delta u_i$ the $i$-directional relative displacements of neighboring two nodes. Again axis-I is normal to crack line and axis-II is normal to crack plane (I–III plane), III axis is parallel to crack line in crack plane. Elastic–plastic values of $J_1$, $J_II$, $J_III$ are related to $K_1$, $K_II$, $K_III$ as Eq. (6).

$$J_1 = \frac{K_1^2}{H}, \quad J_II = \frac{K_II^2}{H}, \quad J_III = \frac{K_III^2}{2G}$$

Here $H = E/(1-\nu)$, $E$ is Young’s modulus. These relations provide ratio $J_1/J_II$ under mode I–II loading and $J_1/J_III$ ratio under mode I–III loading as

$$\frac{J_1}{J_II} = \frac{K_1^2}{K_II^2}, \quad \frac{J_1}{J_III} = \frac{2G}{H} \frac{K_1^2}{K_III^2}$$
The maximum value of \( J \) is the absolute value of \( J \) in each specimen of Fig. 10. The load \( P \) is decomposed into tensile force \( P_n \) and shear force \( P_s \). Further, moment \( M (= P_n l_m) \) due to the eccentric load is also applied on the center of nugget section. Here \( l_m \) is the distance from the center of nugget section to the loading point. In CP–CT tensile mode specimens, tensile force and moment act on the nugget interface. In PS shear mode specimen, only shear force acts on the nugget interface. In TS mixed mode specimen, tensile force, moment and shear force all act together. Note that the initial moment arm in TS specimen is much smaller than that of CP or CT specimen, and it also decreases with the nugget rotation caused by load increase. Moment in TS specimen is consequently negligible compared to tensile and shear forces. Depending on the loading type, the failure of spot-welded specimen takes mode I–II–III mixed aspect. In CP–CT specimens, tensile force \( P_n \) and moment \( M \) dominate the failure, while shear force \( P_s \) dominates the failure in TS specimen. In the 3-folded PS specimen, mode III fracture is experimentally observed at angular position 90° (Fig. 10d).

In the study of fracture criterion and crack growth direction under mixed mode loading, fracture is assumed to occur when a fracture function \( f(K_1, K_{II}, K_{III}) \) reaches its critical value \( f_c \). The functional forms of \( f \) and \( f_c \) are contrived in many ways. First of all, from the viewpoint of energy balance criterion, fracture occurs when total energy release rate, \( G = G_1 + G_{II} \), reaches the critical value. As \( G_1 = K_1^2/H \), \( G_{II} = K_{II}^2/H \) in mode I and II loading, respectively, the following mixed mode fracture criterion can be suggested.

\[
K_1^2 + K_{II}^2 = \text{constant} = K_{IC}^2
\]  

Since \( K_{II} = 0 \) for mode I cracking, \( K_1^2 = K_{IC}^2 \). Since \( K_1 = 0 \) for mode II cracking, \( K_{II}^2 = K_{IC}^2 \). Consequently, Eq. (9) requires \( K_{IC}^2 = K_{IC}^2 \), which means that the locus for combined mode cracking is a circle of radius \( K_{IC} \). In experimental practice, however, \( K_{IC} \neq K_{IC} \). Hence, the fracture criterion would more likely to be (Jurif and Pipes, 1982)

\[
\left( \frac{K_1}{K_{IC}} \right)^2 + \left( \frac{K_{II}}{K_{IC}} \right)^2 = 1
\]

\[\iff K_1^2 + \left( \frac{K_{IC}}{K_{IC}} \right)^2 K_{II}^2 = K_{IC}^2 \]  

The energy balance criterion (9) is based on the assumption that a crack propagates in a self-similar manner. In other words, it is assumed that
crack extension is in the plane of the original crack. In mixed mode experiments, it is usually observed that cracks tend to propagate in the direction slanted from the original cracked plane. This invalidates the assumption. The energy release rate criterion can be modified in such a way that the crack will grow in the direction of maximum energy release rate. Such a criterion can be shown to be equivalent to the maximum tangential stress (MTS) fracture criterion proposed by Erdogan and Sih (1963) The MTS fracture criterion postulates that a crack will grow in the normal direction to the maximum tangential stress. At the crack-tip under mixed mode loading, let \( \theta_m \) be an angular position of maximum tangential stress measured from the original crack plane. The angle \( \theta_m \) is determined from \( \tau_o = 0 \) or equivalently \( \partial \sigma_o / \partial \theta = 0 \) condition. When \( \sigma_o (\theta_m) \) is related to fracture stress, \( \sigma_i = (K_{IC}/\sqrt{2\pi r}) \) of pure mode I loading, the following fracture criterion for mixed mode loading is established.

\[
K_1 \cos^3 \frac{\theta_m}{2} - 3K_{II} \cos^2 \frac{\theta_m}{2} \sin \frac{\theta_m}{2} = K_{IC} \tag{11}
\]

With reference to the functional forms of Eqs. (10) and (11), the \( K_{IC} \) based fracture criterion for mixed mode loading could be generalized as

\[
f_1(K_1^2) + f_2(K_1, K_{II}) + f_3(K_{II}^2) = K_{IC} \tag{12}
\]

Letting \( f_2 = 0 \) in Eq. (12) and revising Eq. (10), Broek (1986) suggested a simplified equivalent stress intensity factor \( K_e \), and corresponding fracture criterion as in Eq. (13). Here \( \beta \) is a material constant, which measures the material sensitivity to mode II loading and is closely related to the ductility of material (Lee et al., 2003).

\[
K_1^2 + \beta K_{II}^2 = K_{IC}^2, \quad \beta = \left( \frac{K_{IC}}{K_{IC}} \right)^2 \tag{13}
\]

It is tough to predict the crack growth direction under mixed mode loading, because the crack does not grow in a self-similar manner, and even once initiated, the crack could keep changing its propagation direction. On this matter, Melin (1987) studied the crack growth direction with a kinked crack model under mixed mode load. He showed that ratio \( K_{H}/K_{IC} \) is a main factor determining the crack growth direction (Lee et al., 2003). Hallback (1997) demonstrated that the fracture angle under mixed mode loading varies with ductility of material. Under pure mode II loading, a crack advances with fracture angle of \(-70^\circ\) in PMMA, but in aluminum alloy, advances with almost 0°, i.e., in the direction predicted by maximum shear stress (MSS) fracture criterion. This indicates that ductility of material is an important factor, which determines the fracture mode. As \( K_{II}/K_{IC} \) is a material constant (= \( \beta^{-0.5} \)) that commands fracture mode, it is related to “material sensitivity to mode II loading”, that is, “ductility of material”. Pook (1971) found a relation, showing that \( K_{IC} \approx 0.75K_{IC} \), through mixed mode fracture tests with aluminum alloy DTD 5050. In such case, material constant \( \beta \) for aluminum DTD 5050 is equal to 1.78.

4.5. Fatigue life prediction with \( J_e \)

In this section, we suggest a \( J_e \)-based fatigue life equation, which is independent of specimen geometry and loading type, for single-spot-welded specimens (CP, CT, TS and PS). We obtained the \( J \)-distribution along the nugget front via elastic-plastic FE analyses as in Fig. 10. The \( J_{max} \) is regarded as the absolute value of vector \( J \) as Eq. (4). The elastic FE displacement solutions produce the modal values \( K_1, K_{II}, K_{III} \) with the crack surface displacement method of Eq. (5). The elastic FE analyses are performed with the same load magnitude with that of elastic-plastic FE analyses above. Substituting \( J_{max} \) and \( K_1, K_{II}, K_{III} \) into Eq. (8) produces the modal values \( J_1, J_{II}, J_{III} \) of \( J \). Extending Eq. (13) into elastic–plastic mixed mode loading, we now define an effective \( J \)-integral, denoted \( J_e \), as Eq. (14)

\[
J_e = J_1 + \beta J_{II} + (1 - \gamma) J_{III} \tag{14}
\]

Here \( \beta \) and \( \gamma \) are material properties associated with fracture mode sensitivity, and \( \nu \) is Poisson’s ratio. The third term \((1 - \nu)\) of Eq. (14) represents \([J_1/J_{III}]/[K_1^2/K_{III}^2] = 1 - \nu \approx 0.7 \) of (14). In mode I CP–CT specimens, \( J_e \) is equal to \( J_1 \), while in mode I and II TS specimen \( J_e \) is determined by \( J_1, J_{II} \) and \( \beta \). In mode I and III PS specimen, \( J_e \) is determined by \( J_1, J_{III} \) and \( \gamma \). Note again that the effective crack driving parameter \( J_e \) of Eq. (14) is
taken as a measure of damage extent at the nugget front, irrespective of crack propagation direction. Lee et al. (2003) recast the load–fatigue life relation into an equivalent-fatigue life relation. In the recasting procedure, they obtained $\beta = 4.3 \quad (= \gamma)$ which minimize the fatigue data scattering. Fig. 13 is the $J_e$-fatigue life relation on SPRC35 rolled steel based on material properties $\beta = \gamma = 4.3$ of SPRC35 rolled steel. Comparison of predicted fatigue lives to experimental ones more than validates the $J_e$-approach. Note that, compared with Fig. 11, the data bandwidth is markedly narrowed in Fig. 13. The $J_e$ concept is similar to the equivalent stress intensity factor (Lee et al., 2003). As based on the elastic–plastic analyses, however, the $J_e$-approach more sharply predicts the fatigue life, especially in higher load cases.

$$J_e = 10^{4.1}N_f^{0.7} \quad (A_1, A_2 = 5.9, -0.5) \quad (15)$$

## 5. Concluding remarks

This study proposed an integrated approach for predicting the fatigue life of spot-weld specimens. FE models embodying the actual behavior were established after the load–deflection curves from overload tests of spot-weld specimens. With the FE models, it was shown that $J$-integral concept alone is insufficient to clearly produce the generalized relationship between load and fatigue life. After discussing the failure mechanism under mixed mode loading, we introduced an effective parameter $J_e$. The recast $J_e$ vs. fatigue life relation successfully demonstrates that Eqs. (14) and (15) can be useful in design and assessment. Followings are the concluding remarks of present study.

(1) The spot-nugget is inhomogeneous due to resistance heat affection. The inhomogeneity should be reflected on FE model to embody the actual mechanical behavior of spot-weld (Figs. 1a and 2a). The FE models of HAZ can be established based on the hardness distribution.

(2) In the CP specimen, ductile fracture in thickness direction occurs close to the nugget, while local thickness necking occurs apart from the nugget in the other specimen. To describe this ductile fracture behavior of the CP specimen, an elastic–plastic porous material needs to be overlapped on HAZ model (Fig. 5).

(3) The fatigue life of spot-weld specimens can be predicted with $J_e$. The $J_e$ describes the effects of specimen geometry and loading type in an inclusive manner. $J_e$-approach of Eq. (15) more sharply defines the fatigue life, especially in higher load cases (Fig. 13).

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## References


