A Spherical Indentation Technique Based on Fea Solutions for Material Property Evaluation

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Abstract. In this work, some inaccuracies and limitation of prior indentation theory, which is based on experimental observations and the deformation theory of plasticity, are first investigated. Then effects of major material properties on the configuration of indentation load-deflection curve are examined via incremental plasticity theory based finite element analyses. A new numerical approach of indentation theory is proposed by examining the finite element solutions at the new optimal point. Numerical regressions of obtained data exhibit that strain hardening exponent and yield strain are the two main parameters which govern the subindenter deformation characteristics. The new indentation theory provides the stress-strain curve and material properties with an average error less than 3%.

Introduction
On this setting the present work begins with a review of prior indentation theory. We then perform finite element (FE) analyses [1]. Comparing FE solutions from deformation and incremental plasticity theories, we clarify the limitation of prior indentation approach based on the deformation plasticity solutions. We then analyze the effects of material properties on the shape of indentation load-depth curve. From incremental plasticity FE solutions, we select an optimal location with the negligible frictional effect. In the present work we examine the FE solutions and propose new numerical indentation formulae for material property evaluation.

Prior spherical indentation theories
Automated indentation test gives stress-strain curve from measured load-depth data. With an indenter of diameter $D$, we have following relation from spherical geometric configuration.

$$d_t = 2\sqrt{h_t D - h_i^2}$$

Here $h_t$ and $d_t$ are ideal indentation depth and projected diameter at loaded state. Tabor [2] came to the experimental conclusion that equivalent (plastic) strain at the indenter contact “edge” is given by

$$\varepsilon_p = 0.2 \left( \frac{d}{D} \right)$$

where $d$ is indented contact diameter. Mean contact pressure $p_m$ is defined by $p_m \equiv 4P/(\pi d^2)$ where $P$ is the compressive indentation load. Then constraint factor $\psi$, which is a function of equivalent plastic strain, is defined as the ratio between mean contact pressure and equivalent stress [3].

$$\psi(\varepsilon_p) \equiv \frac{p_m}{\sigma}$$

Hence, we can express the equivalent stress in the form

\[
\sigma = \frac{4P}{\pi d^2 \psi}
\]  

(4)

\[
\frac{\varepsilon_i}{\varepsilon_o} = \begin{cases} \frac{\sigma}{\sigma_o} & \text{for } \sigma \leq \sigma_o \\ \left(\frac{\sigma}{\sigma_o}\right)^n & \text{for } \sigma > \sigma_o ; \quad 1 < n \leq \infty \end{cases}
\]

(5)

where \(\sigma_o\) is yield strength, \(\varepsilon_o = \sigma_o/E\) yield strain and \(n\) strain hardening exponent. Total strain \(\varepsilon_i\) is decomposed into elastic and plastic strains \((\varepsilon_i = \varepsilon_e + \varepsilon_p)\).

**Finite element modeling and analysis of indentation test**

Figure 1 shows FE model of ball indentation test. We perform nonlinear geometry change FE analyses using isotropic elastic-plastic material, which obeys \(J_2\) flow theory. Considering both loading and geometric symmetries, we use four-node axisymmetric element CAX4 [1]. The lower degree of CAX4 shape function is supplemented by placing fine elements with size 0.0625% of indenter diameter at the material contact surface. FE model consists of about 16400 elements and 17200 nodes. We also place contact surfaces [1] at both material and indenter surfaces. Axisymmetric boundary conditions are imposed on the nodes on axisymmetric axis. The indenter moves down to penetrate the material with the bottom of specimen fixed. The diameter, Young’s modulus and Poisson’s ratio of semi-rigid indenter are 1mm, 537GPa and 0.24 respectively.

**Deformation Characteristics with plasticity theory**

Figure 2 shows equivalent plastic strain distribution along contact surface with deformation and incremental plasticity theories. For a given amount of indentation \((d=0.5D)\), strain from deformation plasticity theory is much smaller than that from incremental plasticity theory, and overall deformation characteristic are quite dissimilar. Tabor’s Eq. (2) for \(d=0.5D\) gives 0.1 as plastic strain at contact edge. But this value 0.1 is far from the incremental plasticity FE solutions. Tabor’s strain value should be interpreted as a subindenter average reference one rather than the accurate one at the contact edge. Sharp strain gradient near contact edge also prohibits accurate measure of “edge” strain.
A New numerical approach

Probing spot for equivalent plastic strain. Taljat et al. [5] selected some data acquisition locations and developed a few sets of indentation formulae based on FEA. However, they neither investigated the frictional effect nor included the variation of yield strain. Being valid only for a specific value of yield strain \( \frac{E}{\sigma_o}=0.002 \) implies that their formula are far from practical use, since yield strain of real materials naturally varies over a wide range. They expressed indentation parameters as functions of only \( n \), and used ideal diameter \( d_i \) instead of actual contact diameter \( d \). Unfortunately, even with these simplified functions of only \( n \), they failed in proposing a definite way of finding the value of \( n \). With these key issues in mind, we probed stress-strain state at subindenter to select a new data acquisition point, which features the negligible friction effect and representative deformation. We first examined the distribution of equivalent plastic strain \( \varepsilon_p \) along radial direction \( r \) at the depth of \( l/D = 0 \) to 50\%, and determined the new reference point. We propose in the present work a new probing depth that is beneath 10\% of indenter diameter from the surface \((l/D=0.1)\) at 0.4\( d \) apart from indentation center. Figures 3-4 show distribution of equivalent plastic strain along \( r \) and \( l \) directions for \( n=10, \, d/D=0.5 \). The proposed point shows negligible contact problem and frictional effect. Moreover, at the probing point, the strain gradient along \( r \) direction is gradual.

New numerical formulae for various material properties. We present a new indentation theory on the basis of FE solutions from the new optimal data acquisition point, \( 2r/d = 0.4 \) and \( l/D = 10\% \). The actual projected contact diameter with pile-up and sink-in in consideration is calculated from the geometric shape of a sphere.

\[
d = 2\sqrt{hD-h^2} = 2\sqrt{c^2h_D-(c^2h_t)^2}
\]

(6)

Here \( h \) is actual indentation depth due to pile-up and sink-in, \( h_t \) is nominal depth measured from the reference surface (= original material surface) and \( c^2 \) is defined as \( c^2 \equiv h/h_t \). Piecewise power law relation (5) for plastic deformation becomes

\[
\sigma = \sigma_o \left( \frac{\varepsilon_i}{\varepsilon_o} \right)^{1/n} = K\varepsilon_i^{1/n}
\]

(7)

where \( K \) is obtained by the regression of stress-strain data. As Eq. (7) is also valid for \( \sigma = \sigma_o \),

\[
\sigma_o = (K^{n/e} / E)^{1/e-1} = E (K/E)^{n/e-1}
\]

(8)
We performed FE analyses first for 13 values of strain hardening exponent with yield strain fixed. We chose an approach of calculating \( d \) from Eq. (6) with \( c^2 \) obtained from regression of FE solutions. Figure 5 shows regression curves of \( c^2 \) against indentation depth for various values of \( n \) with \( f = 0.1 \). Figure 5 reveals \( c^2 \) is a function of indentation depth unlike the outcome of Matthews [6] and Hill et al. [7] in which \( c^2 \) was a constant for a given value of \( n \). Note that even in sufficiently indented fully plastic state, \( c^2 \) keeps slightly increasing, instead of saturating to a constant value, with indentation depth. The FE solutions of \( c^2 \) in Fig. 5 can be expressed with the following equation.

\[
c^2 = f^c_0(n) + f^c_i(n) \ln \left( \frac{h_i}{D} \right) ; \quad f^c_i(n) = a_i n^{-j} ; \quad i = 0, 1, j = 0, 1, 2, 3, 4
\]  

where \( a \) is a coefficient of polynomial function.

Figure 6 shows the FE solutions and corresponding regression curves of equivalent plastic strain against indentation depth for various values of strain hardening exponent \( n \). We observe from the figure that yield strain is the dominant parameter for the regression formula of \( \varepsilon_p \) expressed as a function of \( h_i \) and \( n \).

\[
\varepsilon_p = f_i^{\varepsilon_p} \left( \frac{h_i}{D} \right)^j ; \quad f_i^{\varepsilon_p} (n) = b_{ij} n^{-j} ; \quad i = 0, 1, 2, 3, j = 0, 1, 2, 3, 4
\]  

Figure 7 is the constraint factor \( \psi \) with \( h_i/D \) curve for various values of \( n \). The increase of \( \psi \) with \( n \) is consistent with the results of Matthews [6]. With our new definition of \( \psi \), however, even in fully plastic state, \( \psi \) keeps increasing with \( h_i/D \) instead of saturating to a constant value. The constraint factor \( \psi \) can be given by the following expression.

\[
\psi = \frac{P}{D \sigma} = f_i^{\psi} \left( \frac{h_i}{D} \right)^j ; \quad f_i^{\psi} (n) = c_{ij} n^{-j} ; \quad i = 0, 1, 2, 3, j = 0, 1, 2, 3, 4
\]  

Figure 8 shows the distribution of equivalent plastic strain along the radial direction for three values of yield strength (\( \varepsilon_o = 0.001, 0.002, 0.004 \)) at \( d/D = 0.5 \) with strain hardening exponent (\( n = 10 \)) fixed. In such cases, the value of \( \varepsilon_p \) is observed to decrease with increasing yield strain for a given indentation depth. We observe from the figure that yield strain is the dominant parameter for indentation test. We thus performed FEA of total 728 cases (\( n : 13 \times \sigma_o ; 7 \times E : 8 \)). Equations (12-14) are integrated regression formulae extending Eqs. (9-11) to various values of yield strain.

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**Fig. 5.** Regression curves of \( c^2 \) vs. indentation depth for various values of strain hardening exponent \( n \).

**Fig. 6.** The regression curves of equivalent plastic strain vs. indentation diameter for various values of strain hardening exponent \( n \).
These comparisons for various n and εο as shown in the figures more than validate our new approach.

Fig. 7. The regression lines of constraint factor ψ vs. indentation depth with respect to n.

\[ c^2 = f^r_i(\varepsilon_ο, n) + f^r_j(\varepsilon_ο, n) \ln (h_i / D) \]  
(12)

\[ f^r_i(\varepsilon_ο, n) = a_{ij}(\varepsilon_ο) n^{-j} \quad ; \quad i = 0, 1, j = 0, 1, 2, 3, 4 \]
\[ a_{ij}(\varepsilon_ο) = a_{ijk} \varepsilon_ο^k \quad ; \quad k = 0, 1, 2, 3 \]

\[ \varepsilon_p = f^s_i(\varepsilon_ο, n) \left( \frac{h_i}{D} \right)^{-i} \]  
(13)

\[ f^s_i(\varepsilon_ο, n) = b_{ij}(\varepsilon_ο) n^{-j} \quad ; \quad i = 0, 1, 2, 3, j = 0, 1, 2, 3, 4 \]
\[ b_{ij}(\varepsilon_ο) = b_{ijk} \varepsilon_ο^k \quad ; \quad k = 0, 1, 2, 3 \]

\[ \psi \equiv \frac{P}{D^2 \sigma} = f^y_i(\varepsilon_ο, n) \left( \frac{h_i}{D} \right)^{-i} \]  
(14)

\[ f^y_i(\varepsilon_ο, n) = c_{ij}(\varepsilon_ο) n^{-j} \quad ; \quad i = 0, 1, 2, 3, j = 0, 1, 2, 3, 4 \]
\[ c_{ij}(\varepsilon_ο) = c_{ijk} \varepsilon_ο^k \quad ; \quad k = 0, 1, 2, 3 \]

**Determination of Young’s modulus by indentation test.** Young’s modulus in indentation test is primarily determined by the slope of unloading load-depth curve or by the amount of elastic recovery. Pharr et al. [8] presumed that unloading load-depth curve is nonlinear, and the initial unloading slope of curve has a close relation with Young’s modulus. Here a determining criterion is set up for initial unloading slope. We modified Pharr’s equation using correction coefficient k based on FEA. This observation leads us to

\[ E = \frac{1 - \nu^2}{d / k S - (1 - \nu^2) / E_i} \quad ; \quad k(\varepsilon_ο, n) = g_j(\varepsilon_ο) n^{-j} \quad ; \quad j = 0, 1 \]
\[ g_j = \lambda_{jk} \varepsilon_ο^k \quad ; \quad k = 0, 1, 2 \]  
(15)

**Material property evaluation using a new numerical approach.** Synthesizing the above mentioned arguments, we prepared a program evaluating material properties from indentation load-depth curve.

First, we initially guess values of n and εο. Then, c^2, ε_p, σ and d are calculated from Eqs. (12-14) for each load-depth data point on the load-depth curve. From these, the values of n, K, ε_0 and ε_0 are calculated from stress-strain relation and the Young’s modulus E is computed from Eq. (15) by using slope S. And then updated E, d, c^2, ε_p, σ, n, K, ε_0 and ε_0 are repeatedly calculated until the updated ε_0 and n are converged within the tolerance. Figures 9 compares predicted and real material curves for ε_0 = 0.002. Solid line is the material curve used for FEA, and symbol is the predicted stress-strain curve. These comparisons for various n and ε_0 as shown in the figures more than validate our new approach.
Fig. 9. Comparison of computed stress-strain curves to those given for $\varepsilon_o = 0.002$ [(a) $n = 5$, (b) $n = 10$].

**Concluding Remarks**

Comparison of FE solutions from deformation and incremental plasticity theories clarified the limitation of prior indentation approach. From incremental plasticity based FE solutions, we proposed in this paper a new set of indentation governing equations. A new program was developed to evaluate material properties by using regression formulae of indentation variables $c^2$, $\varepsilon_p$, and $\psi$. We generated load-depth curve by FE analyses for indentation depth of 6\% of indenter diameter once for all. The load-depth curves were converted to stress-strain curves, which provided material properties.

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